Adding and Subtracting Polynomial Functions

5.1

FOCUSING QUESTION How do you add or subtract polynomial functions?

LEARNING OUTCOMES

- I can use tables, graphs, and equations to add polynomials.
- I can use tables, graphs, and equations to subtract polynomials.
- I can apply mathematics to solve problems arising in everyday life, society, and the workplace.

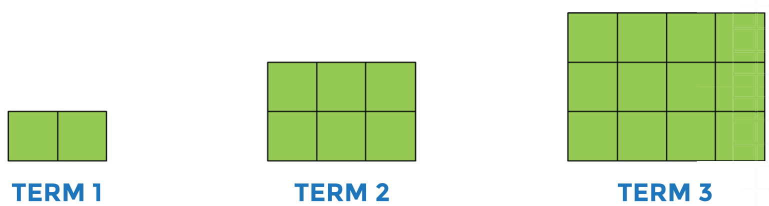
ENGAGE

At a roadside safety rest area near Hillsboro, Texas, the playground includes a shade screen in the shape of a five-pointed star. The interior of the star is a regular pentagon surrounded by congruent isosceles triangles. If the one leg of the isosceles triangles is $3\frac{2}{3}$ yards long, what is the perimeter of the shade screen?

$36\frac{2}{3}$ yards

EXPLORE

A sequence of rectangles can be built using color tiles as shown.



1. Use color tiles to build Term 4 and Term 5.
See margin.

TEKS

AR.4A Connect tabular representations to symbolic representations when adding, subtracting, and multiplying polynomial functions arising from mathematical and real-world situations, such as applications involving surface area and volume.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

2C Learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

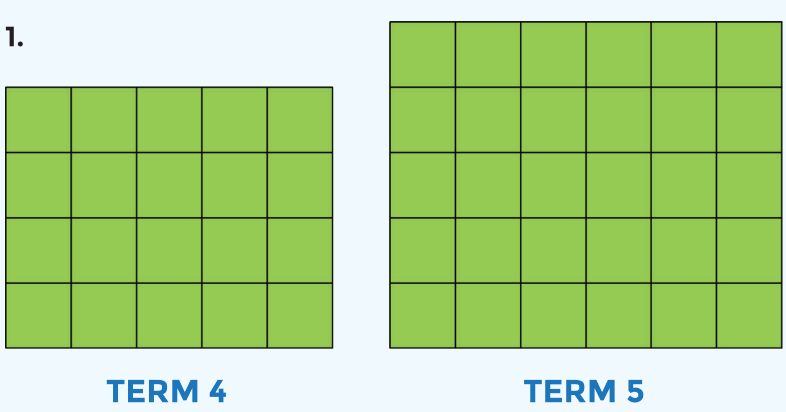
VOCABULARY

polynomial

MATERIALS

- color tiles (50 per group of students)
- graphing technology

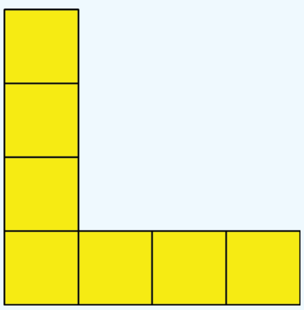
1.



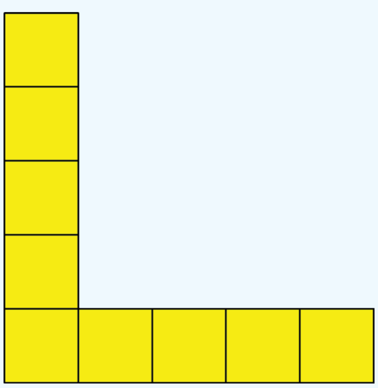
INTEGRATING TECHNOLOGY

You can use tables of function values to see if two function rules are equivalent. Use the function editor of graphing technology to create a table of values for $Y_1 = n(n + 1)$ and $Y_2 = n^2 + n$. For each x-value, if $Y_1 = Y_2$, then the function rules are equivalent.

4.



TERM 4



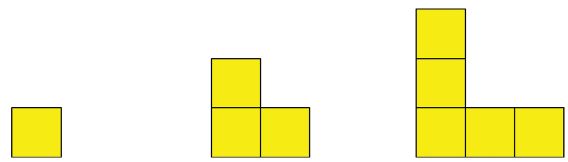
TERM 5

2. Construct a table of values to show the relationship between the term number, n, and the area of the rectangle, $R(n)$. Work backwards in your table to determine the area the rectangle that represents Term 0.

TERM NUMBER, n	AREA OF RECTANGLE, $R(n)$
0	***0***
1	***2***
2	***6***
3	***12***
4	***20***
5	***30***
n	

3. Use patterns in the table, such as finite differences or successive ratios, to determine the function rule, $R(n)$.
$R(n) = n(n + 1) = n^2 + n$

4. A second sequence can be represented as shown. Use color tiles to build Term 4 and Term 5 of this sequence.



TERM 1 **TERM 2** **TERM 3**

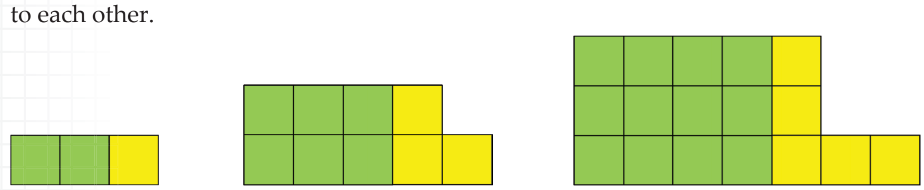
See margin.

5. Add a column to your table for values of $L(n)$, the area of each figure in the second sequence of L-shaped figures. Calculate the area of each figure.

6. Use patterns in the table, such as finite differences or successive ratios, to determine the function rule, $L(n)$.
$L(n) = 1 + 2(n - 1) = 2n - 1$

TERM NUMBER, n	AREA OF RECTANGLE, $R(n)$	AREA OF FIGURE, $L(n)$
0	***0***	***–1***
1	***2***	***1***
2	***6***	***3***
3	***12***	***5***
4	***20***	***7***
5	***30***	***9***
n	$n^2 + n$	

You can create a combined sequence by placing the figures from each sequence next to each other.



TERM 1 **TERM 2** **TERM 3**

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7. Add a column to your table for values of $C(n)$, the combined area of each figure in the combined sequence. Calculate the area of each figure.

TERM NUMBER, n	AREA OF RECTANGLE, $R(n)$	AREA OF FIGURE, $L(n)$	COMBINED AREA, $C(n)$
0	**0**	**–1**	**–1**
1	**2**	**1**	**3**
2	**6**	**3**	**9**
3	**12**	**5**	**17**
4	**20**	**7**	**27**
5	**30**	**9**	**39**
n	$n^2 + n$	$2n - 1$	

8. Use patterns in the table, such as finite differences or successive ratios, to determine the function rule, $C(n)$.
$C(n) = n^2 + 3n - 1$

9. To add polynomials together, you combine like terms. Add the functions for $R(n)$ and $C(n)$ to determine the combined function $C(n) = R(n) + L(n)$. Use the order of operations to simplify the function.
See margin.

10. How does the function rule for $C(n)$ you wrote from number patterns in the table compare to the function rule you wrote for $C(n)$ from the function rules for $R(n)$ and $L(n)$?
The function rules are equivalent.

11. Add a column to your table for values of $D(n)$, the difference between the areas of each figure in the original two sequences. Calculate the differences for each pair of figures.

TERM NUMBER, n	AREA OF RECTANGLE, $R(n)$	AREA OF FIGURE, $L(n)$	COMBINED AREA, $C(n)$	DIFFERENCE IN AREA, $D(n)$
0	**0**	**–1**	**–1**	**1**
1	**2**	**1**	**3**	**1**
2	**6**	**3**	**9**	**3**
3	**12**	**5**	**17**	**7**
4	**20**	**7**	**27**	**13**
5	**30**	**9**	**39**	**21**
n	$n^2 + n$	$2n - 1$	$n^2 + 3n - 1$	

12. Use patterns in the table, such as finite differences or successive ratios, to determine the function rule, $D(n)$.
$D(n) = n^2 - n + 1$

13. To subtract polynomials, you distribute −1 to the second polynomial and then combine like terms. Subtract the functions for $R(n)$ and $L(n)$ to determine the difference function $D(n) = R(n) - L(n)$. Use the order of operations to simplify the function.
See margin.

14. How does the function rule for $D(n)$ you wrote from number patterns in the table compare to the function rule you wrote for $D(n)$ from the function rules for $R(n)$ and $L(n)$?
The function rules are equivalent.

9. $C(n) = (n^2 + n) + (2n - 1)$

$C(n) = n^2 + n + 2n - 1$

$C(n) = n^2 + 3n - 1$

13. $D(n) = (n^2 + n) - (2n - 1)$

$D(n) = n^2 + n - 2n + 1$

$D(n) = n^2 - n + 1$

REFLECT ANSWERS:

The function values for the sum (combined) function were generated by adding the function values for each sequence function together. The function values for the difference function were generated by subtracting the function values for each sequence function.

The operations used to add or subtract two polynomial functions were the same. If you add or subtract from the table, you add or subtract each pair of function values and then use the patterns in the sums or differences to generate a new function rule. If you add or subtract two function rules, then you use the operations to add or subtract polynomials instead of function values. The sum or difference is the function represented in symbolic form.

SUPPORTING ENGLISH LANGUAGE LEARNERS

During classroom discussions and interactions regarding the Reflect questions, make sure to emphasize academic vocabulary in written and spoken forms. English language learners need to see, hear, and say academic vocabulary in a variety of contexts and situations in order to learn academic vocabulary and internalize the words.

REFLECT

- **Discuss with a partner:**
  How did you use addition or subtraction to calculate the function values for the sum (combined) function and difference function in a table?
  See margin.

- **Discuss with a partner:**
  When you add or subtract two polynomial functions, how does using operations on the symbolic functions relate to using operations with table values?
  See margin.

EXPLAIN

ELPS CONNECTION

As you read this section, look for English vocabulary in written classroom materials such as this textbook or other written reference materials you have routinely available in the classroom.

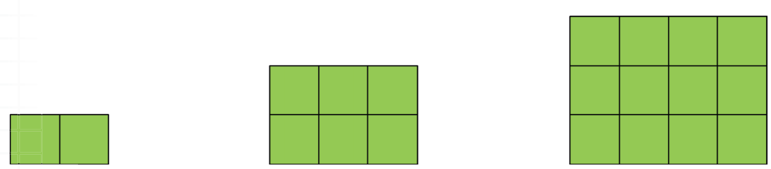
Addition is an operation that allows you to combine two or more quantities. Subtraction is an operation that allows you to separate two or more quantities. You can use tables to add or subtract polynomial functions. You can also represent combined polynomial functions symbolically.

Watch Explain and You Try It Videos

or click here

ADDING POLYNOMIAL FUNCTIONS

Consider the first sequence that you examined in this lesson. This sequence consisted of a series of plans for rectangular flowerbeds.



PLAN 1 **PLAN 2** **PLAN 3**

Let each of these plans represent a rectangular flowerbed that will be created in a city park. The perimeter of the flowerbed is calculated using the formula $P = 2l + 2w$, where l represents the length of the flowerbed and w represents the width of the flowerbed.

You can use a table to represent the two parts of the perimeter formula, $2l$ and $2w$. Since each flowerbed is a rectangle, the perimeter of each flowerbed is the sum of two lengths and two widths.

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PLAN NUMBER, n	LENGTHS, $2l$		WIDTHS, $2w$		PERIMETER, P
1	2(1) = 2	+	2(2) = 4	=	6
2	2(2) = 4	+	2(3) = 6	=	10
3	2(3) = 6	+	2(4) = 8	=	14
4	2(4) = 8	+	2(5) = 10	=	18
5	2(5) = 10	+	2(6) = 12	=	22

Another way to find the perimeters is to combine the symbolic form of the function rules for the length and width. Use finite differences to write function rules for the two lengths, the two widths, and the perimeter. Work backwards in your table to determine the peimeter the rectangle that represents Term 0.

	PLAN NUMBER, n	LENGTHS, $2l$	
	0	0	
+1	1	2(1) = 2	+2
+1	2	2(2) = 4	+2
+1	3	2(3) = 6	+2
+1	4	2(4) = 8	+2
+1	5	2(5) = 10	+2

WIDTHS, $2w$	
2	
2(2) = 4	+2
2(3) = 6	+2
2(4) = 8	+2
2(5) = 10	+2
2(6) = 12	+2

PERIMETER, P	
2	
6	+4
10	+4
14	+4
18	+4
22	+4

- $2l = 2n + 0 = 2n$
- $2w = 2n + 2$
- $P = 4n + 2$

In the table, each individual pair of values for $2l$ and $2w$ are added together to get the perimeter. Then, number patterns (in this case, first differences) are used to generalize a function rule.

However, you can generalize the function rules for each of the addends (in this case, 2l and 2w) and then use the addends to determine the function for the sum (in this case, perimeter, P). The function P(n) can be used to represent the perimeter.

$$P = 2l + 2w$$
$$P(n) = 2n + (2n + 2)$$
$$P(n) = 2n + 2n + 2$$
$$P(n) = 4n + 2$$

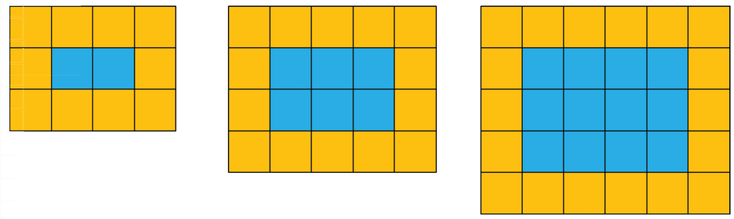
The function rules are equivalent.

INSTRUCTIONAL HINT

This section of the book will use many formulas for perimeter, area, surface area, and volume. Some students may benefit from the use of a standard formula chart. Consider providing one as students recall these formulas from previous mathematics courses.

SUBTRACTING POLYNOMIAL FUNCTIONS

Great Pines Pool Company designs swimming pools. One series has a rectangular-shaped pool surrounded by a tiled walkway.



You can use a table to represent the area of the pool and the combined area of the pool and walkway. From there, the area of the walkway alone can be calculated by subtracting the area of the pool from the combined area.

POOL NUMBER, n	COMBINED AREA OF POOL AND WALKWAY, $C(n)$		AREA OF POOL, $P(n)$		AREA OF WALKWAY, $W(n)$	
1	3(4) = 12	−	1(2) = 2	=	10	
						+4
2	4(5) = 20	−	2(3) = 6	=	14	
						+4
3	5(6) = 30	−	3(4) = 12	=	18	
						+4
4	6(7) = 42	−	4(5) = 20	=	22	
						+4
5	7(8) = 56	−	5(6) = 30	=	26	

You can use finite differences to determine that $W(n) = 4n + 6$.

Another way to separate the areas is to use function rules written in symbolic form. Use finite differences to write function rules for the combined area and the area of the pool, and then use operations on functions to subtract the area of the pool from the combined area. Work backwards in your table to determine the area the rectangle that represents Term 0.

PLAN NUMBER, n	COMBINED AREA OF POOL AND WALKWAY, $C(n)$
0	6
1	3(4) = 12
2	4(5) = 20
3	5(6) = 30
4	6(7) = 42
5	7(8) = 56

Differences in n: +1, +1, +1, +1, +1
First differences in $C(n)$: +6, +8, +10, +12, +14
Second differences in $C(n)$: +2, +2, +2, +2

AREA OF POOL, $P(n)$
0
1(2) = 2
2(3) = 6
3(4) = 12
4(5) = 20
5(6) = 30

First differences in $P(n)$: +2, +4, +6, +8, +10
Second differences in $P(n)$: +2, +2, +2, +2

- $C(n) = (n + 2)(n + 3) = n^2 + 5n + 6$
- $P(n) = n(n + 1) = n^2 + n$

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Once you have the combined area function and area of the pool function, you can determine the area of the walkway, $W(n)$ by subtracting the area of the pool, $P(n)$, from the combined area, $C(n)$.

$$W(n) = C(n) - P(n)$$
$$W(n) = (n^2 + 5n + 6) - (n^2 + n)$$
$$W(n) = n^2 + 5n + 6 - n^2 - n$$
$$W(n) = 4n + 6$$

The function rules are equivalent.

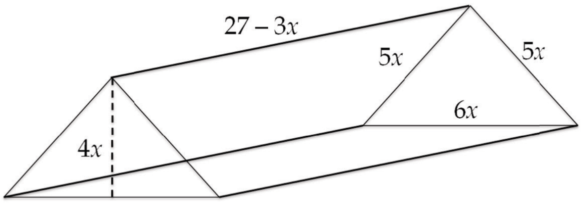
ADDING OR SUBTRACTING POLYNOMIAL FUNCTIONS

Polynomial functions are a group of functions that includes constant functions, linear functions, quadratic functions, and cubic functions.

- You can add polynomial functions together two ways:
  1. Make a table of values for the addends to generate the values of the function that is the sum. Then, use finite differences to generate a symbolic function rule.
  2. Add two polynomial functions together by adding the symbolic function rules and using the properties of algebra to simplify the expressions.
- You can subtract polynomial functions two ways:
  1. Make a table of values for the subtrahend and the minuend to generate the values of the function that is the difference. Then, use finite differences to generate a symbolic function rule.
  2. Subtract two polynomial functions by subtracting the symbolic function rules and using the properties of algebra to simplify the expressions.

EXAMPLE 1

Write polynomial representations for $B(x)$, the area of the base of the triangular prism, $L(x)$, the lateral surface area of the triangular prism, and $T(x)$, the total surface area of the triangular prism. Add the polynomial functions to show that $T(x) = 2B(x) + L(x)$.



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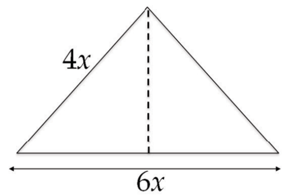
INSTRUCTIONAL HINT

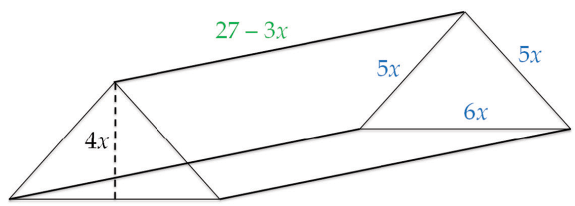
Listen for students to give the incorrect sum for $x = 1$ of $T(x) = 396$. If this happens, have students reread the beginning setup of the example to identify what $T(x)$ represents. $T(x) = 2B(x) + L(x)$ where some students might mistakenly calculate the sum only.

STEP 1 Use the information in the figure to write a function, $B(x)$, which represents the area of the base of the triangular prism.



$$A = \frac{1}{2}lw$$
$$B(x) = \frac{1}{2}(6x)(4x)$$
$$B(x) = \frac{1}{2}(24x^2)$$
$$B(x) = 12x^2$$

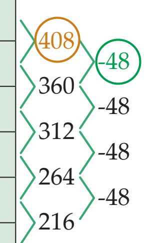
STEP 2 Use the information in the figure to write a function, $L(x)$, which represents the lateral surface area of the triangular prism.



$$L = Ph$$
$$L(x) = (5x + 5x + 6x)(27 - 3x)$$
$$L(x) = (16x)(27 - 3x)$$
$$L(x) = 432x - 48x^2$$
$$L(x) = -48x^2 + 432x$$

STEP 3 Create a table of values for the total surface area of the triangular prism. Use finite differences in the table, including working backwards to find a zero term, to determine $T(x)$, a function that represents the total surface area of the triangular prism.

x	AREA OF THE BASE OF TRIANGULAR PRISM, $B(x)$	LATERAL AREA OF TRIANGULAR PRISM, $L(x)$	TOTAL SURFACE AREA OF TRIANGULAR PRISM, $T(x)$
0			0
1	$12(1)^2 = 12$	$-48(1)^2 + 432(1) = 384$	408
2	$12(2)^2 = 48$	$-48(2)^2 + 432(2) = 672$	768
3	$12(3)^2 = 108$	$-48(3)^2 + 432(3) = 864$	1,080
4	$12(4)^2 = 192$	$-48(4)^2 + 432(4) = 960$	1,344
5	$12(5)^2 = 300$	$-48(5)^2 + 432(5) = 960$	1,560



The second differences are constant. The function is quadratic. Use the patterns in the finite differences to determine a, b, and c, and write the function rule for $T(x)$.

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$2a = -48$ $\quad a + b = 408$ $\quad c = 0$

$a = -24$ $\quad -24 + b = 408$

$b = 432$

Therefore, the function rule is $T(x) = -24x^2 + 432x$.

STEP 4 **Add $2B(x) + L(x)$. Compare the sum to $T(x)$.**

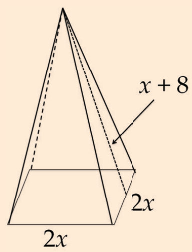
$2B(x) + L(x) = 2[12x^2] + [-48x^2 + 432x]$
$2B(x) + L(x) = 24x^2 - 48x^2 + 432x$
$2B(x) + L(x) = -24x^2 + 432x$

Therefore, $T(x) = 2B(x) + L(x)$.

YOU TRY IT! #1

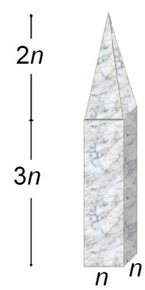
By calculating the sum of the area of the base and the lateral surface area, you can find the surface area of the square pyramid shown below. Write polynomial representations for $B(x)$, the area of the base of the square pyramid, $L(x)$, the lateral surface area of the square pyramid, and $T(x)$, the total surface area of the square pyramid. Add the polynomial functions to show that $T(x) = B(x) + L(x)$.

$B(x) = 4x^2$, $L(x) = 4x^2 + 32x$, and $T(x) = 8x^2 + 32x$



EXAMPLE 2

A stonemason carves similar obelisks from marble with the dimensions shown. Write polynomial functions for the volumes of each portion of the obelisk and the total volume of the obelisk. Use the functions to show that the total volume of marble for an obelisk is equivalent to the sum of the volume of marble in the prism-shaped portion of the obelisk and the volume of marble in the pyramid-shaped portion of the obelisk. If the stonemason has a piece of marble with a volume of 4,000 cubic inches that is at least 10 inches wide and deep, is there enough marble for him to carve an obelisk with a 10-inch base? Justify your answer.

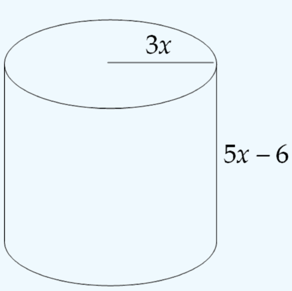


ADDITIONAL EXAMPLE

Using the cylinder given, write polynomial representations for $B(x)$, the area of the base of the cylinder, $L(x)$, the lateral surface area of the cylinder, and $T(x)$, the total surface area of the cylinder. Add the polynomial functions to show that

$T(x) = 2B(x) + L(x)$. Leave all functions in terms of π.

$B(x) = 9\pi x^2$, $L(x) = 30\pi x^2 - 36\pi x$, and $T(x)=48\pi x^2 - 36\pi x$



WIDTH OF BASE, n (INCHES)	VOLUME OF PRISM, $P(n)$ (CUBIC INCHES)	VOLUME OF SQUARE PYRAMID, $S(n)$ (CUBIC INCHES)	TOTAL VOLUME OF OBELISK, $V(n)$ (CUBIC INCHES)
1	(1)(1)(3) = 3	$\left(\frac{1}{3}\right)(1)(1)(2) = \frac{2}{3}$	$3\frac{2}{3}$
2	(2)(2)(6) = 24	$\left(\frac{1}{3}\right)(2)(2)(4) = 5\frac{1}{3}$	$29\frac{1}{3}$
3	(3)(3)(9) = 81	$\left(\frac{1}{3}\right)(3)(3)(6) = 18$	99
4	(4)(4)(12) = 192	$\left(\frac{1}{3}\right)(4)(4)(8) = 42\frac{2}{3}$	$234\frac{2}{3}$
5	(5)(5)(15) = 375	$\left(\frac{1}{3}\right)(5)(5)(10) = 83\frac{1}{3}$	$458\frac{1}{3}$

STEP 1 Use finite differences in the table, including working backwards from the table to find a zero term, to determine $P(n)$, a function that represents the volume of the portion of the obelisk that is a prism.

WIDTH OF BASE, n (INCHES)	VOLUME OF PRISM, $P(n)$ (CUBIC INCHES)
0	0
1	(1)(1)(3) = 3
2	(2)(2)(6) = 24
3	(3)(3)(9) = 81
4	(4)(4)(12) = 192
5	(5)(5)(15) = 375

First differences: 3, 21, 57, 111, 183
Second differences: 18, 36, 54, 72
Third differences: 18, 18, 18

The third differences in the y-values are all 18. $P(n)$ is a cubic function. Use the patterns in the finite differences to determine $P(n)$.

$6a = 18$
$a = 3$

$6a + 2b = 18$
$18 + 2b = 18$
$2b = 0$
$b = 0$

$a + b + c = 3$
$3 + 0 + c = 3$
$3 + c = 3$
$c = 0$

$d = 0$

$P(n) = 3n^3$

STEP 2 Use finite differences in the table, including working backwards to find a zero term, to determine $S(n)$, a function that represents the volume of the top portion of the obelisk that is a square pyramid.

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WIDTH OF BASE, n (INCHES)	VOLUME OF SQUARE PYRAMID, $S(n)$ (CUBIC INCHES)
0	0
1	$\left(\frac{1}{3}\right)(1)(1)(2) = \frac{2}{3}$
2	$\left(\frac{1}{3}\right)(2)(2)(4) = 5\frac{1}{3}$
3	$\left(\frac{1}{3}\right)(3)(3)(6) = 18$
4	$\left(\frac{1}{3}\right)(4)(4)(8) = 42\frac{2}{3}$
5	$\left(\frac{1}{3}\right)(5)(5)(10) = 83\frac{1}{3}$

First differences: $\frac{2}{3}$, $4\frac{2}{3}$, $12\frac{2}{3}$, $24\frac{2}{3}$, $40\frac{2}{3}$

Second differences: 4, 8, 12, 16

Third differences: 4, 4, 4

The third differences in the y-values are all 4. $S(n)$ is a cubic function. Use the patterns in the finite differences to determine $S(n)$.

$6a = 4$ $\quad$ $a = \frac{2}{3}$

$6a + 2b = 4$ $\quad$ $4 + 2b = 4$ $\quad$ $2b = 0$ $\quad$ $b = 0$

$a + b + c = \frac{2}{3}$ $\quad$ $\frac{2}{3} + 0 + c = \frac{2}{3}$ $\quad$ $\frac{2}{3} + c = \frac{2}{3}$ $\quad$ $c = 0$

$d = 0$

$S(n) = \frac{2}{3}n^3$

STEP 3 **Use finite differences in the table, including working backwards to find a zero term, to determine $V(n)$, a function that represents the total volume of the obelisk.**

WIDTH OF BASE, n (INCHES)	TOTAL VOLUME OF OBELISK, $V(n)$ (CUBIC INCHES)
0	0
1	$3\frac{2}{3}$
2	$29\frac{1}{3}$
3	99
4	$234\frac{2}{3}$
5	$458\frac{1}{3}$

First differences: $3\frac{2}{3}$, $25\frac{2}{3}$, $69\frac{2}{3}$, $135\frac{2}{3}$, $223\frac{2}{3}$

Second differences: 22, 44, 66, 88

Third differences: 22, 22, 22

The third differences in the y-values are all 22. $V(n)$ is a cubic function. Use the patterns in the finite differences to determine $V(n)$.

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$6a = 22$ | $6a + 2b = 22$ | $a + b + c = 3\frac{2}{3}$ | $d = 0$

$a = \frac{11}{3}$ | $22 + 2b = 22$ | $3\frac{2}{3} + 0 + c = 3\frac{2}{3}$

$a = 3\frac{2}{3}$ | $2b = 0$ | $3\frac{2}{3} + c = 3\frac{2}{3}$

$b = 0$ | $c = 0$

$V(n) = 3\frac{2}{3}n^3$

STEP 4 Use $P(n)$, $S(n)$, and $V(n)$ to show that $V(n) = P(n) + S(n)$.

$P(n) + S(n) = 3n^3 + \frac{2}{3}n^3 = \frac{9}{3}n^3 + \frac{2}{3}n^3 = \frac{11}{3}n^3 = 3\frac{2}{3}n^3 = V(n)$

Therefore, $V(n) = P(n) + S(n)$.

STEP 5 Determine whether there is enough marble in a block of marble at least 10 inches wide and deep with a total volume of 4,000 cubic inches to carve an obelisk that has a base with sides 10 inches long.

For the obelisk to have a base that has sides 10 inches long, $n = 10$.

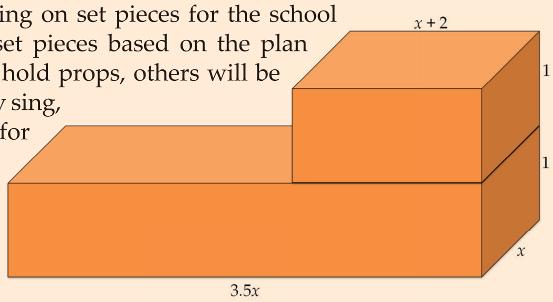
$V(n) = 3\frac{2}{3}n^3$

$V(10) = 3\frac{2}{3}(10)^3 = 3\frac{2}{3}(1{,}000) = 3{,}666\frac{2}{3}$ cubic inches

Since $3{,}666\frac{2}{3} < 4{,}000$, the stonemason has enough marble to carve an obelisk with a base that has sides 10 inches long.

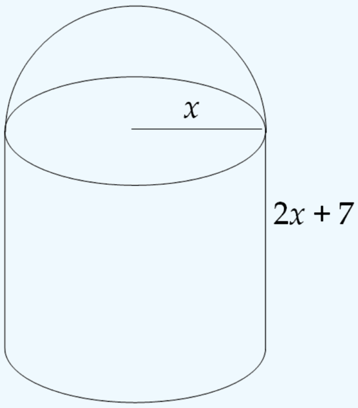
YOU TRY IT! #2

Technical theater students are working on set pieces for the school musical. They plan to use several set pieces based on the plan shown below. Some will be used to hold props, others will be risers for actors to stand on while they sing, and a few will be large raised stages for dancing chorus members. Use the table of values to write polynomial functions for the volume of each set piece in terms of its width x and use the functions you generate to show that the total volume of the set piece is the sum of the volume of each set piece. If two theater technicians work together and one builds a bottom portion that has a volume of 171.5 cubic feet, what will be the volume of the top portion of the set piece that the other theater technician must build? Justify your answer.



ADDITIONAL EXAMPLE

A farmer wants to build a silo like the one shown to store grain. Write polynomial functions, in terms of π, for $H(x)$, the volume of the hemisphere portion of the silo, $C(x)$, the volume of the cylindrical portion of the silo, and $T(x)$, the total volume of the silo. If the farmer typically stores about 2,500 cubic feet of grain each year, what should the radius of his silo be to the nearest foot?



$H(x) = \frac{2}{3}\pi x^3$, $C(x) = 2\pi x^3 + 7\pi x^2$, and $T(x) = \frac{8}{3}\pi x^3 + 7\pi x^2$

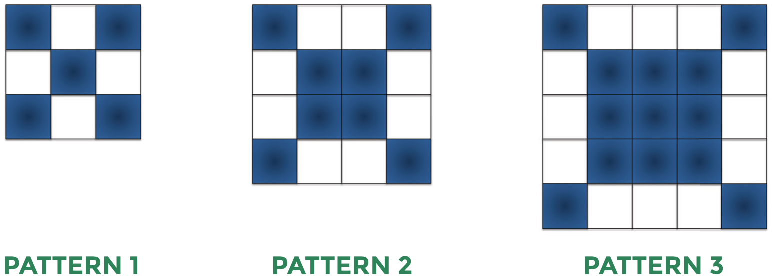
The farmer should build his silo with a radius of 6 feet.

WIDTH OF SET PIECE, x (FEET)	VOLUME OF BOTTOM PORTION, $B(x)$ (CUBIC FEET)	VOLUME OF TOP PORTION, $T(x)$ (CUBIC FEET)	TOTAL VOLUME OF SET PIECE, $V(x)$ (CUBIC FEET)
1	(1)(1)(3.5) = 3.5	(1)(1)(3) = 3	6.5
2	(1)(2)(7) = 14	(1)(2)(4) = 8	22
3	(1)(3)(10.5) = 31.5	(1)(3)(5) = 15	46.5
4	(1)(4)(14) = 56	(1)(4)(6) = 24	80
5	(1)(5)(17.5) = 87.5	(1)(5)(7) = 35	122.5

See margin.

EXAMPLE 3

A contractor creates custom patterns for tile backsplashes. Write polynomials to describe how many tiles are needed for each pattern and how many blue tiles are needed for each pattern. Use these to write a polynomial to express how many white tiles are needed for the nth pattern.



STEP 1 **Use the geometric pattern to determine $T(n)$, a function that represents the total number of tiles needed for the n^{th} tile backsplash pattern.**

Each pattern is a square with side lengths that have two more tiles than the pattern number. Therefore, $T(n) = (n + 2)^2$.

STEP 2 **Use the geometric pattern to determine $B(n)$, a function that represents the number of blue tiles needed for the n^{th} tile backsplash pattern.**

Each pattern has a square of blue tiles in the middle of the pattern as well as blue tiles in the four corners of the larger square. Therefore, $B(n) = n^2 + 4$.

YOU TRY IT! #2 ANSWER:

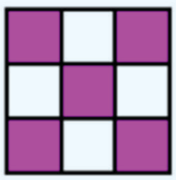
$B(x) = 3.5x^2$, $T(x) = x^2 + 2x$, and $V(x) = 4.5x^2 + 2x$.

$B(x) + T(x) = 3.5x^2 + x^2 + 2x = 4.5x^2 + 2x = V(x)$; therefore, $V(x) = B(x) + T(x)$.

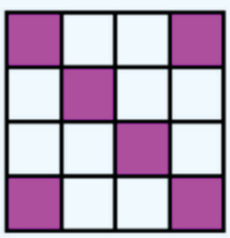
The volume of the top portion of the set piece that the other theater technician must build is 63 cubic feet.

ADDITIONAL EXAMPLE

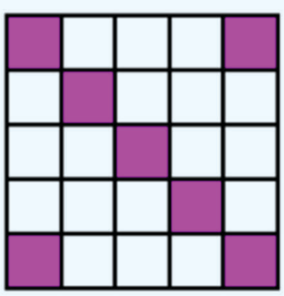
Joanna is making a quilt for her niece. She lays out various size options for the quilt using the same pattern. Write polynomials to describe how many squares of fabric are needed for each quilt option and how many pink squares are needed for each quilt. Use these to write a polynomial rule to express how many white squares are needed for the n^{th} quilt pattern.



Quilt 1



Quilt 2



Quilt 3

$T(n) = (n + 2)^2$ and $P(n) = n + 4$

The number of white squares required for the nth quilt pattern is $W(n) = x^2 + 3x$.

STEP 3 **Use $T(n)$ and $B(n)$ to determine $W(n)$, a function that represents the number of white tiles needed for the n^{th} tile backsplash pattern.**

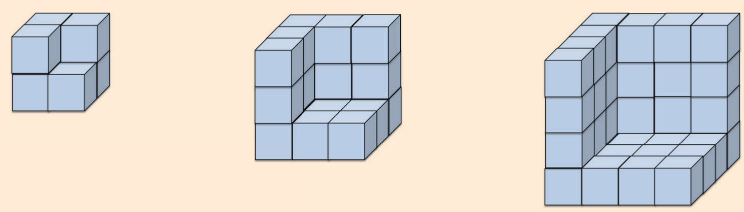
The number of white tiles needed for the tile backsplash pattern can be found by subtracting the number of black tiles needed from the total number of tiles needed. Therefore, $W(n) = T(n) - B(n)$.

$W(n) = T(n) - B(n) = (n + 2)^2 - (n^2 + 4) = n^2 + 4n + 4 - n^2 - 4 = 4n$

The geometric pattern also confirms that $W(n) = 4n$ since each of the four sides of the larger square contains as many white tiles as the pattern number.

YOU TRY IT! #3

Shannon builds cubes using sets of blocks. Her little sister removes a smaller cube from each set to create the new sets of blocks shown below. Use a table to generate function values for the first three sets. Write polynomials to describe the number of blocks required for each set of Shannon's original design and the number of blocks her little sister removed. Use the polynomials you generated to write a polynomial to describe how many blocks will be required for the nth new set of blocks.



SET 1 **SET 2** **SET 3**

The number of cubes required for the nth new set of blocks is $3n^2 + 3n + 1$.

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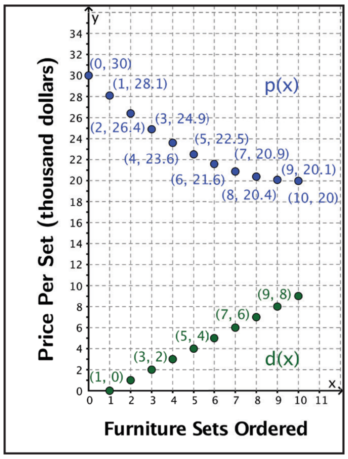
EXAMPLE 4

The graph shows $p(x)$, the price per set of a customer's order of office furniture sets versus the number of the office furniture sets the customer ordered and $d(x)$, the discounted amount for bulk ordering. The customer's price per set is discounted for bulk ordering.



Write polynomial functions for $p(x)$ and $d(x)$. Use these functions to generate $n(x)$, the new price per item of a customer's order after the discount is applied. If a customer orders a dozen sets of office furniture, how much will each set cost after the discount is applied? Justify your answer.

STEP 1 Construct a table from some of the points on the graph of $p(x)$. Use finite differences in the table to determine $p(x)$, a function that represents the original price per set of an order of x sets of office furniture.

NUMBER OF SETS ORDERED, x	ORIGINAL PRICE PER SET, $p(x)$ (THOUSAND DOLLARS)
0	30
1	28.1
2	26.4
3	24.9
4	23.6
5	22.5

First differences: -1.9, -1.7, -1.5, -1.3, -1.1

Second differences: 0.2, 0.2, 0.2, 0.2

The second differences are constant. The function is quadratic. Use the patterns in the finite differences to determine a, b, and c, and write the function rule for $p(x)$.

$2a = 0.2$
$a = 0.1$

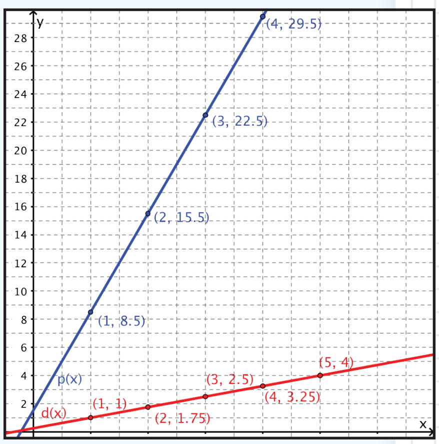
$a + b = -1.9$
$0.1 + b = -1.9$
$b = -2$

$c = 30$

Therefore, the function rule is $p(x) = 0.1x^2 - 2x + 30$.

ADDITIONAL EXAMPLE

The local pizza parlor is offering discounts on pizza orders in the month of January. The more pizzas a customer buys, the higher the discount will be. The price for x pizzas, $p(x)$, is shown in the graph along with the discount for x pizzas, $d(x)$. Write polynomial functions for $p(x)$ and $d(x)$. Then use these functions to generate $n(x)$, the new price of a customer's order after the discount is applied. If a customer orders 10 pizzas, how much will her order cost?



$p(x) = 7x + 1.50$,
$d(x) = 0.75x + 0.25$,
and $n(x) = 6.25x + 1.25$

The order for 10 pizzas will cost \$63.75.

STEP 2 **Construct a table, including working backwards from the table to find a zero term, from some of the points on the graph of $d(x)$. Use finite differences in the table to determine $d(x)$, a function that represents the discount per set of an order of x sets of office furniture.**

NUMBER OF SETS ORDERED, x	DISCOUNT PER SET, $d(x)$ (THOUSAND DOLLARS)
0	0 - 1 = -1
1	0
2	1
3	2
4	3
5	4

First differences: 1, 1, 1, 1, 1

The first differences are constant. The function is linear. Use the patterns in the finite differences to determine a and b, and write the function rule for $d(x)$.

$a = 1$ $b = -1$

Therefore, the function rule is $d(x) = x - 1$.

STEP 3 **Use $p(x)$ and $d(x)$ to determine $n(x)$, a function that represents the new price per item of a customer's order after the discount is applied.**

Discounts are applied by subtracting the discount from the original price. Therefore, $n(x) = p(x) - d(x)$.

$$n(x) = p(x) - d(x) = (0.1x^2 - 2x + 30) - (x - 1) = 0.1x^2 - 2x + 30 - x + 1$$
$$n(x) = 0.1x^2 - 2x - x + 30 + 1 = 0.1x^2 - 3x + 31$$

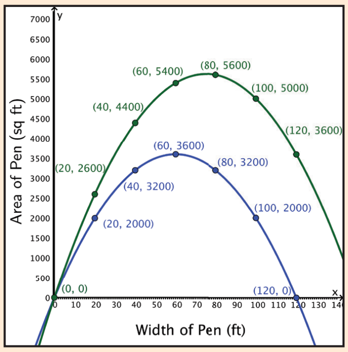
STEP 4 **Use $n(x)$ to determine the cost per set to a customer who orders a dozen sets of office furniture.**

$$n(x) = 0.1x^2 - 3x + 31$$
$$n(12) = 0.1(12)^2 - 3(12) + 31 = 0.1(144) - 36 + 31 = 14.4 - 36 + 31 = -21.6 + 31$$
$$n(12) = 9.4$$

A customer who orders a dozen sets of office furniture will pay \$9,400 per set of office furniture once the discount is applied.

YOU TRY IT! #4

Two farmers create rectangular pens for their livestock. Farmer Blue has 240 linear feet of fencing for his pen and Farmer Green has 300 linear feet of fencing for his pen. The areas of their pens are shown in the graph below, with the blue graph representing Farmer Blue's pen and the green graph representing Farmer Green's pen. Write a polynomial to express how much more space Farmer Green's livestock will have than Farmer Blue's. If both farmers construct rectangular pens with widths of 75 feet, how much more space will Farmer Green's livestock have than Farmer Blue's livestock?



Farmer Green's livestock will have 2250 square feet more area than Farmer Blue's livestock.

PRACTICE/HOMEWORK

For questions 1 – 4, use the table below.

x	$g(x)$	$h(x)$	$f(x)$
0	3	0	9
1	8	9	33
2	23	36	105
3	48	81	225
4	83	144	393
5	128	225	609

1. Determine the function rule for $g(x)$.
$g(x) = 5x^2 + 3$

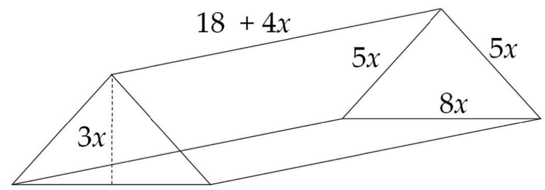
2. Determine the function rule for $h(x)$.
$h(x) = 9x^2$

3. Determine the function rule for $f(x)$.
$f(x) = 24x^2 + 9$

4. Use $g(x)$, $h(x)$, and $f(x)$ to determine whether $f(x) = 3g(x) + h(x)$.
Yes, $f(x) = 3g(x) + h(x)$

Use the diagram of the triangular prism to complete questions 5 – 7.

GEOMETRY



5. Complete the table to show the area of the base, the lateral area, and the total surface area of the triangular prism above, for various values of x.

x	AREA OF THE BASE, $B(x)$	LATERAL AREA, $L(x)$	TOTAL SURFACE AREA, $T(x)$
0	$\left(\frac{1}{2}\right)(0)(0) = 0$	(0 + 0 + 0)(18 + 0) = 0	0
1	$\left(\frac{1}{2}\right)(8)(3) = 12$	(5 + 5 + 8)(18 + 4) = 396	420
2	$\left(\frac{1}{2}\right)(16)(6) = 48$	(10 + 10 + 16)(18 + 8) = 936	1032
3	***108***	***1620***	1836
4	***192***	***2448***	2832
5	***300***	***3420***	4020

6. Write polynomial representations for $B(x)$, the area of the base of the triangular prism, $L(x)$, the lateral surface area of the triangular prism, and $T(x)$, the total surface area of the triangular prism.
$B(x) = 12x^2$, $L(x) = 72x^2 + 324x$, $T(x) = 96x^2 + 324x$

7. Use your function representations to verify that $T(x) = 2B(x) + L(x)$.
See margin.

7. *$T(x) = 2B(x) + L(x)$*

$T(x) = 2(12x^2) + (72x^2 + 324x) = 24x^2 + 72x^2 + 324x$

$T(x) = 96x^2 + 324x$

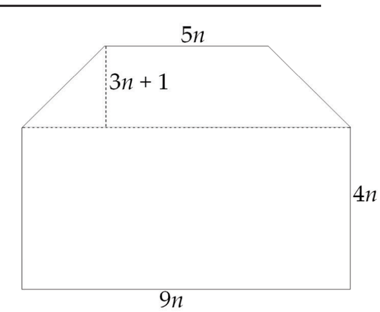
Therefore, $T(x) = 2B(x) + L(x)$.

Use the information below to complete problems 8 – 10.

ART AND ARCHITECTURE

Gabe is painting a wall in his house. It is shaped like a rectangle with a trapezoid on top.

The table shows the area of each part of the wall, and the total area, for various values of n.



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x	AREA OF THE RECTANGLE, $R(n)$ (SQUARE FEET)	AREA OF THE TRAPEZOID, $T(n)$ (SQUARE FEET)	TOTAL SURFACE AREA, $A(n)$ (SQUARE FEET)
1	(9)(4) = 36	$\left(\frac{1}{2}\right)(14)(4) = 28$	64
2	(18)(8) = 144	$\left(\frac{1}{2}\right)(28)(7) = 98$	242
3	(27)(12) = 324	$\left(\frac{1}{2}\right)(42)(10) = 210$	534
4	(36)(16) = 576	$\left(\frac{1}{2}\right)(56)(13) = 364$	940
5	(45)(20) = 900	$\left(\frac{1}{2}\right)(70)(16) = 560$	1460

8. Use the table to write polynomial functions for the area of each part of the wall, and the total area.
$R(n) = 36n^2$, $T(n) = 21n^2 + 7n$, $A(n) = 57n^2 + 7n$

9. Use your functions to verify that the total area is equivalent to the sum of each part of the wall.
See margin.

9. $A(n) = R(n) + T(n)$

$A(n) = 36n^2 + (21n^2 + 7n)$

$A(n) = 57n^2 + 7n$

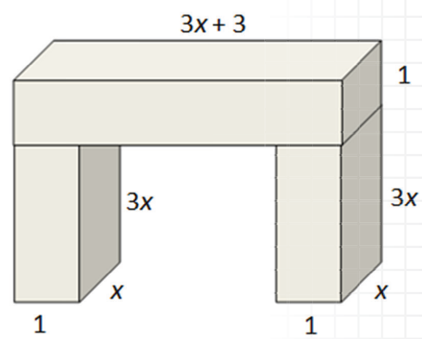
Therefore,
$A(n) = R(n) + T(n)$

10. If the value of n is 2.5 feet, what is the total area of the wall?
373.75 square feet

Use the information below to complete questions 11 – 13.

GEOMETRY

Melissa and Kyle are making a bench for their yard. They plan to use several wood boxes, as shown below.



11. Use the diagram to write polynomial functions for the volume of each leg, $L(x)$, the volume of the top piece of the bench, $T(x)$, and the total volume of the bench, $V(x)$.
$L(x) = 3x^2$, $T(x) = 3x^2 + 3x$, $V(x) = 9x^2 + 3x$

12. Use the functions you generated to show that the total volume of the bench is the sum of the volume of its parts.
See margin.

12. $V(x) = 2L(x) + T(x)$

$V(x) = 2(3x^2) + (3x^2 + 3x)$
$= 6x^2 + 3x^2 + 3x = 9x^2 + 3x$

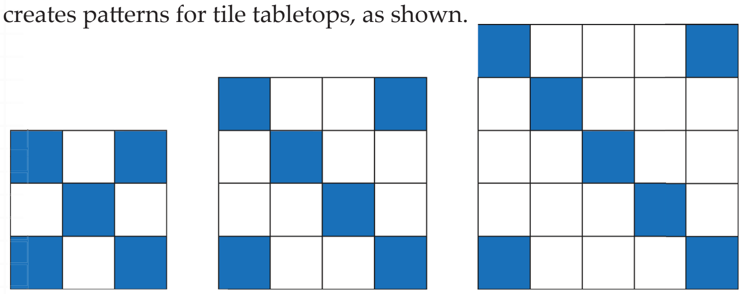
Therefore,
$V(x) = 2L(x) + T(x)$.

13. If Melissa and Kyle build a bench leg that has a volume of 0.75 cubic feet, what will be the total volume of the bench?
$V(x) = 3.75$ cubic feet

Use the information below to complete questions 14 – 16.

ART AND ARCHITECTURE

Ramon creates patterns for tile tabletops, as shown.



PATTERN 1 **PATTERN 2** **PATTERN 3**

14. Use a table to generate function values, look for patterns, and write polynomial functions that describe $T(n)$ the number of total tiles needed for each pattern, and $B(n)$, the number of blue tiles needed for each pattern.
$T(n) = (n + 2)^2$, $B(n) = n + 4$

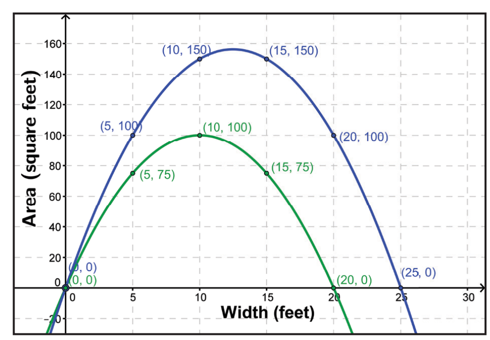
15. Use a table to write a polynomial function, $W(n)$, that describes the number of white tiles needed for the nth pattern.
$W(n) = n^2 + 3n$

16. How many white tiles will Ramon need for the 10th pattern?
130 white tiles

Use the information below to complete questions 17 – 20.

CRITICAL THINKING

Grant and Bianca are each building a rectangular sandbox for their kids. Each one plans to make the box 1 foot high, but they have different amounts of wood to construct their box. Grant has 40 linear feet of wood, while Bianca has 50 linear feet.



17. Notice the possible areas of each box shown on the graph, with the green graph representing Grant's box, and the blue graph representing Bianca's box. What is the area of Bianca's sandbox when its width is 5 feet?
100 cubic feet

18. Write a polynomial function to express the difference, $d(x)$ in area between Bianca's sandbox and Grant's sandbox.
$d(x) = 5x$

19. If Grant and Bianca both construct rectangular sandboxes with widths of 12.5 feet, how much more space will Bianca's sandbox have than Grant's sandbox?
62.5 cubic feet more

20. If Grant and Bianca both construct rectangular sandboxes with the same widths, and Bianca's sandbox has 55 cubic feet more than Grant's, what is the width of each of sandbox?
11 feet

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