

Study Guide and Intervention

Composing Functions

Example

A wrecking ball with a mass of 2000 kg is released from the top of a 40-meter crane. Using its height at any time before hitting the concrete below, calculate the ball's gravitational potential energy. Use the function $h(t) = -4.9t^2 + 40$ to determine the height of the wrecking ball as it falls. Use the function $p(h) = m \cdot g \cdot h$ to calculate its potential energy, where mass $m = 2,000$ kg, gravity $g = 9.8$ meters per second squared, and the height $h(t)$ is in meters.

Solution

STEP 1 Create a table for $h(t) = -4.9t^2 + 40$ in 0.5 second intervals.

TIME (SECONDS), t	0	0.5	1	1.5	2	2.5	3
HEIGHT (METERS), $h(t)$	40	38.775	35.1	28.975	20.4	9.375	-4.1

STEP 2 Create a table for the potential energy $p(h) = m \cdot g \cdot h$ of the wrecking ball from the time it is released from the crane until it hits the concrete below.

HEIGHT (METERS), h	40	38.775	35.1	28.975	20.4	9.375	-4.1
POTENTIAL ENERGY (JOULES), $p(h)$	784,000	759,990	687,960	567,910	399,840	183,750	-80,360

STEP 3 Combine the tables with the domain for $h(t) = -4.9t^2 + 40$ and the range for $p(h) = m \cdot g \cdot h$ to reflect potential energy with respect to time.

TIME (SECONDS), t	0	0.5	1	1.5	2	2.5	3
POTENTIAL ENERGY (JOULES), $p(t)$	784,000	759,990	687,960	567,910	399,840	183,750	-80,360

STEP 4 Write the potential energy function as a composition of the functions for height and potential energy.

$$p(h) = m \cdot g \cdot h$$

$$p(h(t)) = 2000 \cdot 9.8 \cdot (-4.9t^2 + 40)$$

$$p(h(t)) = 19,600(-4.9t^2 + 40)$$

$$p(h(t)) = -96,040t^2 + 784,000$$

$p(h(t))$ can also be written as $(p \circ h)(t)$.

Exercises

For questions 1 - 6 use the scenario below.

A local department store is having a clearance sale and discounting everything in the store by 25%. The sales tax rate is 8.25%.

1. Write a function $f(x)$ to represent the discounted amount of a purchase, x .

2. Use the function $f(x)$ to complete the table below.

ORIGINAL AMOUNT OF PURCHASE, IN DOLLARS, x	50.00	75.00	100.00	125.00	150.00
DISCOUNTED AMOUNT OF PURCHASE, IN DOLLARS, $f(x)$					

3. Write a function $g(x)$ to represent the amount of sales tax charged on any purchase, x .

4. Use the function $g(x)$ to complete the table below. Round to the nearest cent.

AMOUNT OF PURCHASE, IN DOLLARS, x	37.50	56.25	75.00	93.75	112.50
AMOUNT OF SALES TAX, IN DOLLARS, $g(x)$					

5. Write a composition of the functions that can be used to calculate the amount of tax charged on a discounted purchase. Simplify the composition.

6. Use the composition $g(f(x))$ to calculate the amount of sales tax on the discounted amount of purchase, based on the original cost of the purchase.

ORIGINAL AMOUNT OF PURCHASE, IN DOLLARS, x	50.00	75.00	100.00	125.00	150.00
AMOUNT OF SALES TAX, IN DOLLARS, $g(x)$					

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Composing Functions (cont.)

Exercises

For questions 7 – 14 use the situation below.

The Metro Motorcycle Company is having a year-end sale on five models of motorcycles. They are offering a rebate of \$2,000 on each motorcycle sold or a 10% discount on each motorcycle sold.

7. Write a function $R(x)$ to represent the cost of a motorcycle, x , after the rebate.

8. Use $R(x)$ to fill in the table of values.

MODEL	A	B	C	D	E
ORIGINAL COST, IN DOLLARS, x	10,000	15,000	20,000	25,000	30,000
COST AFTER REBATE, IN DOLLARS, $R(x)$					

9. Write a function $D(x)$ to represent the cost of a motorcycle, x , after the 10% discount.

10. Use $D(x)$ to fill in the table of values.

MODEL	A	B	C	D	E
ORIGINAL COST, IN DOLLARS, x	10,000	15,000	20,000	25,000	30,000
COST AFTER DISCOUNT, IN DOLLARS, $D(x)$					

11. The Metro Motorcycle Company decides to give customers the rebate and the discount on all models of motorcycles. Write the composed function if the company applies the rebate first, and then the discount.

12. Fill in the table of values using $D(R(x))$.

MODEL	A	B	C	D	E
ORIGINAL COST, IN DOLLARS, x	10,000	15,000	20,000	25,000	30,000
REDUCED COST, IN DOLLARS, $D(R(x))$					

13. After several weeks, The Metro Motorcycle Company decides to change the way it calculates the reduced price of its motorcycles by applying the discount first, and then the rebate. Write the composed function representing this situation.

14. Fill in the table of values using $R(D(x))$.

MODEL	A	B	C	D	E
ORIGINAL COST, IN DOLLARS, x	10,000	15,000	20,000	25,000	30,000
REDUCED COST, IN DOLLARS, $R(D(x))$					