Logarithmic Functions

1. Generate the logarithmic function that is the inverse of the exponential function
   $g(x) = (10)^x - 2$

   SOLUTION:

$x = (10)^y - 2$	switch x and y
$x + 2 = (10)^y - 2 + 2$	add 2 to both sides
$\log (x + 2) = \log (10)^y$	take the log of both sides
$\log (x+ 2) = y$	

   ANSWER:

   $g^{-1}(x) = \log (x + 2)$

2. Generate the logarithmic function that is the inverse of the exponential function
   $g(x) = (10)^{x+1} + 3$

   SOLUTION:

$x = (10)^{y+1} + 3$	switch x and y
$x - 3 = (10)^{y+1} + 3 - 3$	subtract 3 from both sides
$\log (x - 3) = \log (10)^{y+1}$	take the log of both sides
$\log (x - 3) - 1 = y + 1 - 1$	subtract 1 from both sides
$\log (x - 3) - 1 = y$	

   ANSWER:

   $g^{-1}(x) = \log (x - 3) - 1$

3. Generate the logarithmic function that is the inverse of the exponential $g(x) = 0.5(10)^x - 1$

   SOLUTION:

$x = 0.5(10)^y - 1$	switch x and y
$x + 1 = 0.5(10)^y - 1 + 1$	add 1 to both sides
$2 * (x + 1) = 2 * 0.5(10)^y$	multiply both sides by 2
$\log (2x + 2) = \log (10)^y$	take the log of both sides
$\log (2x + 2) = y$	

   ANSWER:

   $g^{-1}(x) = \log (2x + 2)$

4. Generate the logarithmic function that is the inverse of the exponential $g(x) = (e)^{2x} + 4$.

   SOLUTION:

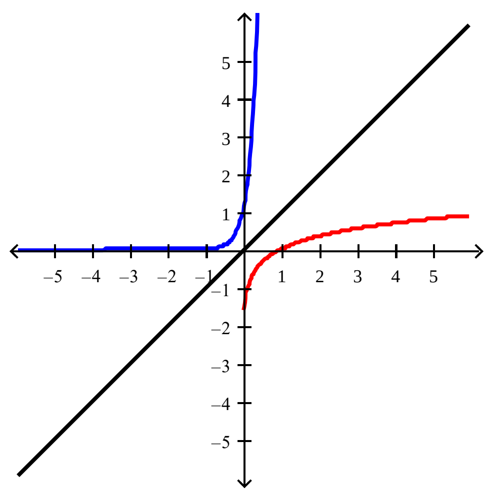
$x = (e)^{2y} + 4$	switch x and y
$x - 4 = (e)^{2y} + 4 - 4$	subtract 4 from both sides
$\ln (x - 4) = \ln (e)^{2y}$	take the ln of both sides
$\ln (x - 4) / 2 = 2y / 2$	divide both sides by 2
$0.5 \ln (x - 4) = y$	

   ANSWER:

   $g^{-1}(x) = 0.5 \ln (x - 4)$

5. Using graphs and tables, verify whether or not $f(x) = 10^{2x}$ and $g(x) = 0.5 \log(x)$ are inverses, including checking any necessary domain or range restrictions.

   SOLUTION:

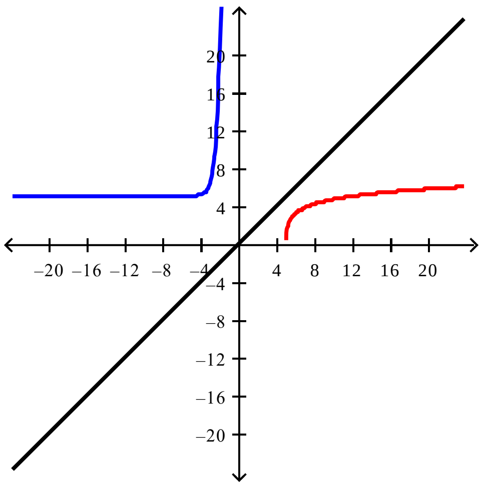


   ANSWER:

   Inverses

6. Using graphs and tables, verify whether or not $f(x) = 10^{x+3} + 5$ and $g(x) = \log(x - 5) + 3$ are inverses, including checking any necessary domain or range restrictions.

SOLUTION:

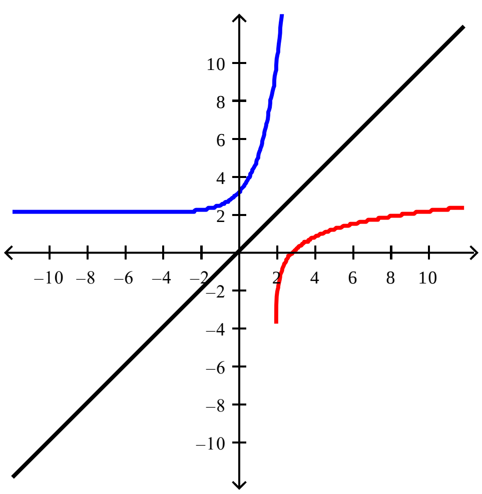


ANSWER:

Not inverses

7. Using graphs and tables, verify whether or not $f(x) = e^x + 2$ and $g(x) = \ln(x - 2)$ are inverses, including checking any necessary domain or range restrictions.

SOLUTION:

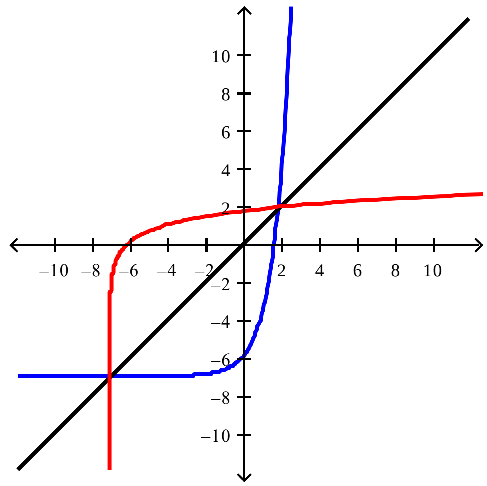


ANSWER:

Inverses

8. Using graphs and tables, verify whether or not $f(x) = 10^{0.5x} - 7$ and $g(x) = 2\log(x + 7)$ are inverses, including checking any necessary domain or range restrictions.

SOLUTION:

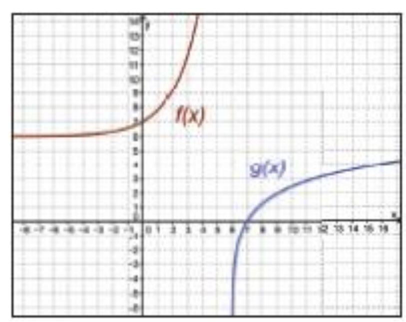


ANSWER:

Inverses

For problems 9 and 10, compare the domain and range as well as any intercepts, if they exist, of the functions graphed below. Write domain and range as inequalities, intervals, or in set builder notation, and determine whether or not $f(x)$ and $g(x)$ are inverses.

9.



SOLUTION:

The domain for f(x) is (-∞, ∞); the range is restricted by a horizontal asymptote, y = 6. The range is (6, ∞)

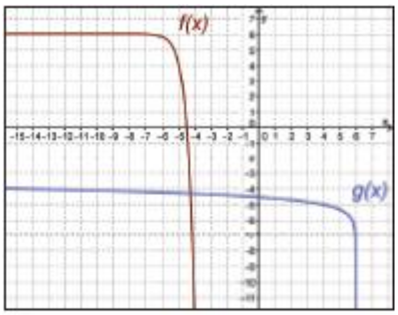
The domain for g(x) is restricted by a vertical asymptote, x = 6. The domain is (6, ∞) and the range is (-∞, ∞)

f(x) has no x-intercept, but it has a y-intercept at (0, 7). g(x) has an x-intercept at (7, 0), but has no y-intercept.

ANSWER:

	f(x)	g(x)
Domain:	(-∞, ∞)	(6, ∞)
Range:	(6, ∞)	(-∞, ∞)
x-intercept:	none	(7, 0)
y-intercept:	(0, 7)	none

10.



SOLUTION:

The domain for f(x) is (-∞, ∞); the range is restricted by a horizontal asymptote, y = 6. The range is (-∞, 6)

The domain for g(x) is restricted by a vertical asymptote, x = 6. The domain is (-∞, 6) and the range is (-∞, ∞)

f(x) has no y-intercept, but it has a x-intercept around (-4.5, 0). g(x) has a y-intercept around (0, -4.5), but has no x-intercept.

ANSWER:

	f(x)	g(x)
Domain:	(-∞, ∞)	(-∞, 6)
Range:	(-∞, 6)	(-∞, ∞)
x-intercept:	(-4.5, 0)	none
y-intercept:	none	(0, -4.5)