

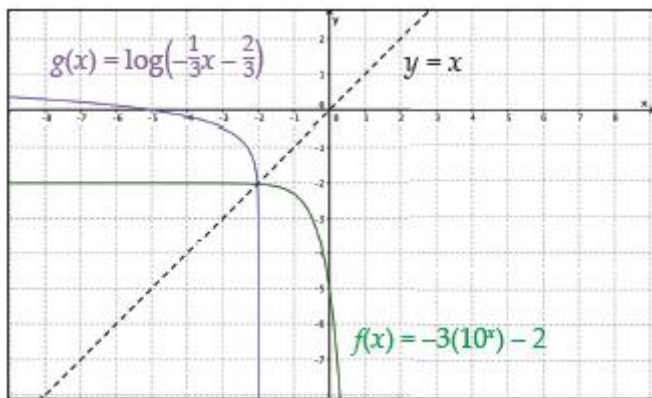
Study Guide and Intervention

Logarithmic Functions

Example Using graphs and tables, verify whether or not $f(x) = -3(10^x) - 2$ and $g(x) = \log(-\frac{1}{3}x - \frac{2}{3})$ are inverses, including checking any domain restrictions.

Solution

Step 1 Graph $f(x)$ and $g(x)$ on the same coordinate plane. Use the line of reflection $y = x$ to determine whether the functions could be inverses.



The graphs appear to be reflections of one another over the line $y = x$.

Step 2 Examine the tables side by side to determine whether the functions could be inverses.

x	$f(x)$
-2	-2.03
-1	-2.3
0	-5
1	-32

x	$g(x)$
-32	1
-5	0
-2	undef.

Step 3 Check for domain and range restrictions for the functions.

Exponential functions have a horizontal asymptote at $y = d$. The range of $f(x)$ is restricted to $y < -2$. The domain of $g(x)$, which is the range of $f(x)$, exhibits the same restriction to $x < -2$.

Yes, the functions are inverses. The y-intercept of $f(x)$, $(0, -5)$, and the x-intercept of $g(x)$, $(-5, 0)$ are inverses. The pairs $(1, -32)$ and $(-32, 1)$ are also inverses. Points above and below those are restricted by asymptotes. Moreover, the graphs of the functions appear to be reflections of one another over the line $y = x$.

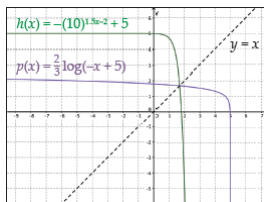
Exercises

- Generate the logarithmic function that is the inverse of the exponential function $g(x) = (10)^x - 2$
- Generate the logarithmic function that is the inverse of the exponential function $g(x) = (10)^{x+1} + 3$
- Generate the logarithmic function that is the inverse of the exponential $g(x) = 0.5(10)^x - 1$
- Generate the logarithmic function that is the inverse of the exponential $g(x) = (e)^{2x} + 4$.
- Using graphs and tables, verify whether or not $f(x) = 10^{2x}$ and $g(x) = 0.5 \log(x)$ are inverses, including checking any necessary domain or range restrictions.
- Using graphs and tables, verify whether or not $f(x) = 10^{0.5x} - 7$ and $g(x) = 2 \log(x + 7)$ are inverses, including checking any necessary domain or range restrictions.
- Using graphs and tables, verify whether or not $f(x) = e^x + 2$ and $g(x) = \ln(x - 2)$ are inverses, including checking any necessary domain or range restrictions.
- Using graphs and tables, verify whether or not $f(x) = 10^{x+3} + 5$ and $g(x) = \log(x - 5) + 3$ are inverses, including checking any necessary domain or range restrictions.

Study Guide and Intervention

Logarithmic Functions (cont.)

Example Compare the domain and range as well as any intercepts, if they exist, of the functions graphed below. Write domain and range as inequalities, intervals, or in set builder notation, and determine whether or not $h(x)$ and $p(x)$ are inverses.



Solution

Step 1 Determine the domain and range of the exponential function $h(x)$.

Since $h(x)$ is an exponential function, its domain contains all real numbers.

The range is restricted by the horizontal asymptote, $y = 5$. Because $a = -1$ in this function, the function is reflected over the x -axis, so the range is all numbers less than 5.

Step 2 Determine the domain and range of the logarithmic function $p(x)$.

Since $p(x)$ is a logarithmic function, its domain is restricted by the vertical asymptote located at the line $x = 5$. Thus, the domain is all numbers less than 5.

The range of $p(x)$ is all real numbers.

Step 3 Compare the domains and ranges of the two functions.

The range of $h(x)$ is the same as the domain of $p(x)$, and the domain of $h(x)$ is the same as the range of $p(x)$.

Step 4 Determine the x - and y -intercepts of $h(x)$ and $p(x)$.

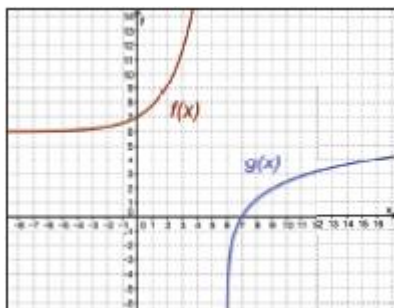
Examining the graph, the x -intercept of $h(x)$ is approximately $(2, 0)$ and its y -intercept is approximately $(0, 5)$. The x -intercept of $p(x)$ is approximately $(5, 0)$ and its y -intercept is approximately $(0, 2)$.

These functions are inverses of one another. The domain and range of $h(x)$ are switched for $p(x)$. Their x - and y -intercepts are inverses of one another.

Exercises

Compare the domain and range as well as any intercepts, if they exist, of the functions graphed below. Write domain and range as inequalities, intervals, or in set builder notation, and determine whether or not $h(x)$ and $p(x)$ are inverses.

9.



10.

