

The image features a blue background with a subtle circuit board pattern. A central black rectangular box with rounded corners contains the text "TRANSFORMING AND ANALYZING EXPONENTIAL FUNCTIONS" in white, bold, uppercase letters. The text is centered within the box and the background has a slight gradient from light to dark blue.

# TRANSFORMING AND ANALYZING EXPONENTIAL FUNCTIONS

# EXPONENTIAL FUNCTIONS

- An exponential function is a function that uses a constant multiplier, or base, to show either growth or decay.
- For an exponential function, the general form is  $f(x) = a(b)^{kx-c} + d$ , where  $a$ ,  $k$ ,  $c$ , and  $d$  are real numbers.
- The exponential parent function is  $f(x) = b^x$ .
- The full family of exponential functions is generated by applying transformations to the exponential parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

# CHANGES IN A

- The parameter  $a$  influences the vertical stretch or compression of the graph.
- If  $|a| > 1$ , then the  $y$ -values are multiplied by a factor of  $a$  to vertically stretch the graph
- If  $0 < |a| < 1$ , then the  $y$ -values are multiplied by a factor of  $a$  to vertically compress the graph
- If  $a < 0$ , then the graph will be reflected across the  $x$ -axis

# CHANGES IN K

- The parameter  $k$  influences the horizontal stretch or compression of the graph.
- If  $|k| > 1$ , then the  $x$ -values are multiplied by a factor of  $\frac{1}{|k|}$  to horizontally compress the graph
- If  $0 < |k| < 1$ , then the  $x$ -values are multiplied by a factor of  $\frac{1}{|k|}$  to horizontally stretch the graph
- If  $k < 0$ , then the graph will be reflected across the  $y$ -axis

## CHANGES IN C

- The parameter  $c$ , like  $k$ , influences the horizontal translation of the graph.
- Note that in the general form, the sign in front of the  $c$  is negative. This means that when reading the value of  $c$  from the equation, you should read the opposite sign from what is given in the equation.
- If  $c > 0$ , then the graph will translate  $|\frac{c}{k}|$  to the right.
- If  $c < 0$ , then the graph will translate  $|\frac{c}{k}|$  to the left.

## CHANGES IN $D$

- The parameter  $d$  influences the vertical translation of the graph.
- If  $d > 0$ , then the graph will translate  $|d|$  units up.
- If  $d < 0$ , then the graph will translate  $|d|$  units down.



# ASYMPTOTES

- Each exponential function has one horizontal asymptote
- The horizontal asymptote is governed by vertical parameters changes. A vertical translation moves the asymptote  $d$  units and a vertical dilation does not move the asymptote.
  - horizontal asymptote:  $y = d$

# DOMAIN AND RANGE

- An exponential function does not have any domain restrictions. Therefore, the domain will always be *all real numbers*, or  $\{x \mid x \in \mathbb{R}\}$
- The range is restricted by the horizontal asymptote,  $y = d$ . If  $a > 0$ , then the range is  $y > d$ . If  $a < 0$ , then the range is  $y < d$ .
  - $a > 0, \{y \mid y > d\}$
  - $a < 0, \{y \mid y < d\}$



# X- AND Y-INTERCEPTS

- An exponential function has at most one x-intercepts. Use the graph and the calculator to determine the value of the x-intercept
- An exponential function has at most one y-intercepts. If it exists, the y-intercept is located at:

- $(0, \frac{a}{b^c} + d)$

## EXAMPLES

- What transformations of the exponential parent function,  $f(x) = 10^x$ , will result in the graph of the exponential function  $g(x) = -3(10)^{2x-1} + 5$ ?

# EXAMPLES

- Step 1: Determine the values of the parameters  $a$ ,  $k$ ,  $c$ , and  $d$  of  $g(x)$  and the value of  $b$ , the base of  $g(x)$ .
  - $a = -3$ ,  $k = 2$ ,  $c = 1$ , and  $d = 5$

## EXAMPLES

- Use the values of the parameters to describe the transformations of the exponential parent function  $f(x)$  that are necessary to produce  $g(x)$ .
- $a = -3$ ; vertical stretch by a factor of 3, reflected over the  $x$ -axis
- $k = 2$ ; horizontal compression by a factor of  $\frac{1}{2}$
- $c = 1$ ; horizontal shift  $\frac{1}{2}$  unit to the right
- $d = 5$ ; vertical shift 5 units up

## EXAMPLES

- Identify the key attributes of  $y = -2^{1.5x-3} + 1$ , including domain and range, asymptote, x-intercept, and y-intercept. Write the domain and range in set builder notation.

# EXAMPLES

- Step 1: Determine the domain, range and asymptote of  $y = -2^{1.5x-3} + 1$ .
  - The domain is all real numbers;  $\{x \mid x \in \mathbb{R}\}$
  - The range is affected by a and d; a is negative,  $d = 1$ 
    - The range is numbers  $< 1$ ;  $\{y \mid y < 1\}$
    - Asymptote:  $y = 1$



## EXAMPLES

- Step 2: Determine if the function has an x-intercept
  - The function has an x-intercept at  $(2, 0)$
- Step 3: Determine if the function has a y-intercept
  - The function has a y-intercept at  $(0, \frac{7}{8})$