TRANSFORMING AND ANALYZING EXPONENTIAL FUNCTIONS

EXPONENTIAL FUNCTIONS

- An exponential function is a function that uses a constant multiplier, or base, to show either growth or decay.
- For an exponential function, the general form is $f(x) = a(b)^{kx-c} + d$, where a, k, c, and d are real numbers.
- The exponential parent function is $f(x) = b^x$.
- The full family of exponential functions is generated by applying transformations to the exponential parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

CHANGES IN *A*

- The parameter *a* influences the vertical stretch or compression of the graph.
- If $|a| > 1$, then the y-values are multiplied by a factor of *a* to vertically stretch the graph
- If $0 < |a| < 1$, then the y-values are multiplied by a factor of *a* to vertically compress the graph
- If $a < 0$, then the graph will be reflected across the x-axis

CHANGES IN K

- The parameter k influences the horizontal stretch or compression of the graph.
- If $|k| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|k|}$ to horizontally compress the graph
- If $0 < |k| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|k|}$ to horizontally stretch the graph
- If $k < 0$, then the graph will be reflected across the y-axis

CHANGES IN C

- The parameter c, like k, influences the horizontal translation of the graph.
- Note that in the general form, the sign in front of the c is negative. This means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $|\frac{c}{k}|$ to the right.
- If $c < 0$, then the graph will translate $|\frac{c}{k}|$ to the left.

CHANGES IN *D*

- The parameter d influences the vertical translation of the graph.
- If $d > 0$, then the graph will translate $|d|$ units up.
- If $d < 0$, then the graph will translate $|d|$ units down.

ASYMPTOTES

- Each exponential function has one horizontal asymptote
- The horizontal asymptote is governed by vertical parameters changes. A vertical translation moves the asymptote d units and a vertical dilation does not move the asymptote.
    - horizontal asymptote: $y = d$

DOMAIN AND RANGE

- A exponential function does not have any domain restrictions. Therefore, the domain will always be *all real numbers*, or $\{x \mid x \in \mathbb{R}\}$
- The range is restricted by the horizontal asymptote, $y = d$. If $a > 0$, then the range is $y > d$. If $a < 0$, then the range is $y < d$.
    - $a > 0, \{y \mid y > d\}$
    - $a < 0, \{y \mid y < d\}$

X- AND Y-INTERCEPTS

- An exponential function has at most one x-intercepts. Use the graph and the calculator to determine the value of the x-intercept
- An exponential function has at most one y-intercepts. If it exists, the y-intercept is located at:
    - $(0, \frac{a}{b^c} + d)$

EXAMPLES

- What transformations of the exponential parent function, $f(x) = 10^x$, will result in the graph of the exponential function $g(x) = -3(10)^{2x-1} + 5$?

EXAMPLES

- Step 1: Determine the values of the parameters a, k, c, and d of g(x) and the value of b, the base of g(x).
  - $a = -3$, $k = 2$, $c = 1$, and $d = 5$

EXAMPLES

- Use the values of the parameters to describe the transformations of the exponential parent function f(x) that are necessary to produce g(x).
- $a = -3$; vertical stretch by a factor of 3, reflected over the x-axis
- $k = 2$; horizontal compression by a factor of $\frac{1}{2}$
- $c = 1$; horizontal shift $\frac{1}{2}$ unit to the right
- $d = 5$; vertical shift 5 units up

EXAMPLES

- Identify the key attributes of $y = -2^{1.5x-3} + 1$, including domain and range, asymptote, x-intercept, and y-intercept. Write the domain and range in set builder notation.

EXAMPLES

- Step 1: Determine the domain, range and asymptote of $y = -2^{1.5x-3} + 1$.
  - The domain is all real numbers; $\{x \mid x \in \mathbb{R}\}$
  - The range is affected by a and d; a is negative, d = 1
  - The range is numbers < 1; $\{y \mid y \leq 1\}$
  - Asymptote: y = 1

EXAMPLES

- Step 2: Determine if the function has an x-intercept
  - The function has an x-intercept at (2, 0)
- Step 3: Determine if the function has a y-intercept
  - The function has a y-intercept at $(0, \frac{7}{8})$