# TRANSFORMING AND ANALYZING RATIONAL FUNCTIONS

# Rational Functions

- A rational function is a function composed of  a ratio of two polynomial functions, p(x) and q(x).
    - r(x) = $\frac{p(x)}{q(x)}$
- For a rational function, the general form is f(x) = $\frac{a}{bx-c}$ + d, where a, b, c, and d are real numbers.
- The rational parent function is f(x) = $\frac{1}{x}$. This is also called an inverse variation function.
- The full family of rational functions is generated by applying transformations to the Rational parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

# Changes in *a*

- The parameter *a* influences the vertical stretch or compression of the graph of the rational function.
- If $|a| > 1$, then the y-values are multiplied by a factor of *a* to vertically stretch the graph
- If $0 < |a| < 1$, then the y-values are multiplied by a factor of *a* to vertically compress the graph
- If $a < 0$, then the graph will be reflected across the x-axis

# Changes in *b*

- The parameter *b* influences the horizontal stretch or compression of the graph of the rational function.
- If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph
- If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph
- If $b < 0$, then the graph will be reflected across the y-axis

# Changes in c

- The parameter *c*, like *b*, influences the horizontal translation of the graph of the rational function.
- Note that in the general form, the sign in front of the *c* is negative. This means that when reading the value of *c* from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $\left|\frac{c}{b}\right|$ to the right.
- If $c < 0$, then the graph will translate $\left|\frac{c}{b}\right|$ to the left.

# Changes in *d*

- The parameter *d* influences the vertical translation of the graph of the rational function.
- If $d > 0$, then the graph of the rational function will translate $|d|$ units up.
- If $d < 0$, then the graph of the rational function will translate $|d|$ units down.

# Asymptotes

- Each rational function has two asymptotes: one horizontal and one vertical
- The horizontal asymptote is governed by vertical parameters changes. A vertical translation moves the asymptote d units and a vertical dilation does not move the asymptote.
- The vertical asymptote is governed by horizontal parameters changes. A horizontal translation moves the asymptote c units and a horizontal dilation moves the asymptote closer or away from the x-axis by a factor of $\frac{1}{b}$.
    - horizontal asymptote: y = d
    - vertical asymptote: x = $\frac{c}{b}$

# Domain and Range

- A rational function involves the ratio of two polynomial functions. Since the function in the denominator can never equal 0, any values of x that cause the denominator to equal 0 are excluded from the domain. Therefore, the domain will always be *all real numbers minus* $\frac{c}{b}$ , or $\{x \mid x \in \mathbb{R}, x \neq \frac{c}{b}\}$

- The range is restricted by the horizontal asymptote, y = d. Therefore, y = d must be excluded from the range of a rational function. $\{y \mid y \in \mathbb{R}, y \neq d\}$

# X- and Y-intercepts

- A rational function has at most one x-intercepts. If it exists, the x-intercept is located at:
  - $(\frac{cd-a}{bd}, 0)$
- The values of b and  d may not equal 0. When b = 0, the function does not exist since there is no x-term in the function. When d = 0, the x-axis is an asymptote and the x-intercept does not exist.

- A rational function has at most one y-intercepts. If it exists, the y-intercept is located at:
  - $(0, \frac{a}{-c} + d)$
- When c = 0, the y-axis is an asymptote and they-intercept does not exist.

# Examples

- What transformations of the rational parent function, f(x) = $\frac{1}{x}$, will result in the graph of the rational function g(x) = - $\frac{3}{x-2}$ + 1.5?

# Examples

- Step 1: Rewrite the equation of g(x) in general form y = $\frac{a}{bx-c}$ + d to determine the values of the parameters a, b, c, and d.
  - g(x) = $\frac{a}{bx-c}$ + d
  - g(x) = - $\frac{3}{x-2}$ + 1.5
  - g(x) = $\frac{-3}{1x-2}$ + 1.5
  - a = -3, b = 1, c = 2, d = 1.5

# Examples

- Step 2: Use the parameters to describe the transformations of the rational parent function f(x) that are necessary to produce g(x).
- a = -3, so there is vertical stretch by a factor of 3; a is negative, so it is reflected over the x-axis
- b = 1, there is no change to the graph
- c = 2, so there is a horizontal shift $\frac{2}{1}$ = 2 units to the right
- d = 1.5, so there is a vertical shift 1.5 units up

# Examples

- Identify the key attributes of y = - $\frac{12}{3x}$ + 2, including domain and range (including asymptotes), x- and y-intercepts. Write the domain and range in set builder notation.

# Examples

- Step 1: Determine the domain and range of y = - $\frac{12}{3x}$ + 2
  - The domain excludes $\frac{c}{b}$
    - $\frac{c}{b} = \frac{0}{3}$ = 0
    - {x | x ∈ ℝ, x ≠ 0}
  - The range excludes d
    - d = 2
    - {y | y ∈ ℝ, y ≠ 2}

# Examples

- Step 2: Determine if the function has an x-intercept.
- Use the calculator to determine the x-intercept.
    - x-int: (-2, 0)

- Step 3: Determine if the function has a y-intercept.
- Use the calculator to determine the y-intercept.
    - y-int: none