TRANSFORMING AND ANALYZING RATIONAL FUNCTIONS

Rational Functions

○ A rational function is a function composed of a ratio of two polynomial functions, p(x) and q(x).

  ○ $r(x) = \frac{p(x)}{q(x)}$

○ For a rational function, the general form is f(x) $= \frac{a}{bx-} + d$, where a, b, c, and d are real numbers.

○ The rational parent function is f(x) $= \frac{1}{x}$. This is also called an inverse variation function.

○ The full family of rational functions is generated by applying transformations to the Rational parent function

○ Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

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Changes in *a*

○ The parameter *a* influences the vertical stretch or compression of the graph of the rational function.

○ If |*a*| > 1, then the y-values are multiplied by a factor of *a* to vertically stretch the graph

○ If 0 < |*a*| < 1, then the y-values are multiplied by a factor of *a* to vertically compress the graph

○ If a < 0, then the graph will be reflected across the x-axis

Changes in *b*

◦ The parameter *b* influences the horizontal stretch or compression of the graph of the rational function.

◦ If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph

◦ If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph

◦ If $b < 0$, then the graph will be reflected across the y-axis

Changes in *c*

◦ The parameter *c*, like *b*, influences the horizontal translation of the graph of the rational function.

◦ Note that in the general form, the sign in front of the *c* is negative. This means that when reading the value of *c* from the equation, you should read the opposite sign from what is given in the equation.

◦ If $c > 0$, then the graph will translate $\left|\frac{c}{b}\right|$ to the right.

◦ If $c < 0$, then the graph will translate $\left|\frac{c}{b}\right|$ to the left.

Changes in *d*

◦ The parameter *d* influences the vertical translation of the graph of the rational function.

◦ If $d > 0$, then the graph of the rational function will translate $|d|$ units up.

◦ If $d < 0$, then the graph of the rational function will translate $|d|$ units down.

Asymptotes

- Each rational function has two asymptotes: one horizontal and one vertical
- The horizontal asymptote is governed by vertical parameters changes. A vertical translation moves the asymptote d units and a vertical dilation does not move the asymptote.
- The vertical asymptote is governed by horizontal parameters changes. A horizontal translation moves the asymptote c units and a horizontal dilation moves the asymptote closer or away from the x-axis by a factor of $\frac{1}{b}$.

  - horizontal asymptote: y = d
  - vertical asymptote: $x = \frac{c}{b}$

Domain and Range

- A rational function involves the ratio of two polynomial functions. Since the function in the denominator can never equal 0, any values of x that cause the denominator to equal 0 are excluded from the domain. Therefore, the domain will always be *all real numbers minus* $\frac{c}{b}$, or {x | x ∈ ℝ, x ≠ $\frac{c}{b}$}

- The range is restricted by the horizontal asymptote, y = d. Therefore, y = d must be excluded from the range of a rational function. {y | y ∈ ℝ, y ≠ d}

X- and Y-intercepts

- A rational function has at most one x-intercepts. If it exists, the x-intercept is located at:

  - $(\frac{cd-a}{bd}, 0)$
- The values of b and d may not equal 0. When b = 0, the function does not exist since there is no x-term in the function. When d = 0, the x-axis is an asymptote and the x-intercept does not exist.

- A rational function has at most one y-intercepts. If it exists, the y-intercept is located at:

  - $(0, \frac{a}{-c} + d)$
- When c = 0, the y-axis is an asymptote and they-intercept does not exist.

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Examples

◦ What transformations of the rational parent function, $f(x) = \frac{1}{x}$, will result in the graph of the rational function $g(x) = -\frac{3}{x-2} + 1.5$?

Examples

◦ Step 1: Rewrite the equation of g(x) in general form $y = \frac{a}{bx-c} + d$ to determine the values of the parameters a, b, c, and d.

◦ $g(x) = \frac{a}{bx-c} + d$

◦ $g(x) = -\frac{3}{x-2} + 1.5$

◦ $g(x) = \frac{-3}{1x-2} + 1.5$

◦ a = -3, b = 1, c = 2, d = 1.5

Examples

◦ Step 2: Use the parameters to describe the transformations of the rational parent function f(x) that are necessary to produce g(x).

◦ a = -3, so there is vertical stretch by a factor of 3; a is negative, so it is reflected over the x-axis

◦ b = 1, there is no change to the graph

◦ c = 2, so there is a horizontal shift $\frac{2}{1} = 2$ units to the right

◦ d = 1.5, so there is a vertical shift 1.5 units up

Examples

- Identify the key attributes of $y = -\frac{12}{3x} + 2$, including domain and range (including asymptotes), x- and y-intercepts. Write the domain and range in set builder notation.

Examples

- Step 1: Determine the domain and range of $y = -\frac{12}{3x} + 2$
    - The domain excludes $\frac{c}{b}$
        - $\frac{c}{b} = \frac{0}{3} = 0$
        - $\{x \mid x \in \mathbb{R}, x \neq 0\}$
    - The range excludes d
        - $d = 2$
        - $\{y \mid y \in \mathbb{R}, y \neq 2\}$

Examples

- Step 2: Determine if the function has an x-intercept.
- Use the calculator to determine the x-intercept.
    - x-int: (-2, 0)

- Step 3: Determine if the function has a y-intercept.
- Use the calculator to determine the y-intercept.
    - y-int: none