Transforming and Analyzing Absolute Value Functions

Absolute Value Functions

- For an Absolute Value Value function, the general form is $f(x) = a|bx - c| + d$, where a, b, c, and d are real numbers.
- The Absolute Value parent function is $f(x) = |x|$
- The full family of Absolute Value functions is generated by applying transformations to the Absolute Value parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

Changes in a

- The parameter a influences the vertical stretch or compression of the graph.
- If $|a| > 1$, then the y-values are multiplied by a factor of a to vertically stretch the graph
- If $0 < |a| < 1$, then the y-values are multiplied by a factor of a to vertically compress the graph
- If $a < 0$, then the graph will be reflected across the x-axis

Changes in *b*

- The parameter *b* influences the horizontal stretch or compression of the graph.
- If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph
- If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph
- If $b < 0$, then the graph will be reflected across the y-axis

Changes in *c*

- The parameter *c*, like *b*, influences the horizontal translation of the graph.
- Note that in the general form, the sign in front of the *c* is negative. This means that when reading the value of *c* from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $|\frac{c}{b}|$ to the right.
- If $c < 0$, then the graph will translate $|\frac{c}{b}|$ to the left.

Changes in d

- The parameter d influences the vertical translation of the graph.
- If $d > 0$, then the graph will translate $|d|$ units up.
- If $d < 0$, then the graph will translate $|d|$ units down.

Vertex

- The vertex of an absolute value graph is a maximum or minimum value.
- If the graph opens up, then the vertex is a minimum value. If the graph opens down, then the vertex is a maximum value.
- The location of the vertex is
  - $(\frac{c}{b}, d)$

Domain and Range

- A Absolute Value function involves squaring a number. Since every real number can be squared, there are no domain restrictions. Therefore, the domain will always be *all real numbers*, or $\{x \mid x \in \mathbb{R}\}$

- The range does have restrictions. The range is affected by parameters a and d. If $a > 0$, then d sets the y-coordinate of the vertex at a minimum value. The range becomes $y \geq d$ or $\{f(x) \mid f(x) \geq d\}$

- If $a < 0$, then d sets the y-coordinate of the vertex at a maximum value. The range becomes $y \leq d$ or $\{f(x) \mid f(x) \leq d\}$

X- and Y-intercepts

- A Absolute Value function has as many as two x-intercepts, also called *zeroes*. The x-intercepts are located at:
  - $\left(\frac{c \pm \frac{d}{a}}{b}, 0\right)$
- If the is in the general form, y = a|bx – c| + d then we find the y-intercept by substituting x = 0:
  - the y-intercept becomes (0, a|c| + d)

Examples

- What transformations of the absolute value parent function, f(x) = |x|, will result in the graph of the absolute value function $g(x) = -\frac{1}{2}|2x + 1| - 3$?

Examples

- Step 1: Rewrite the equation of g(x) in general form to determine the values of the parameters a, b, c, and d.
  - $g(x) = a(bx - c)^2 + d$
  - $g(x) = \frac{1}{3}(x - 1)^2 - 4$
  - $g(x) = \frac{1}{3}(x - 1)^2 + (-4)$
  - So, $a = \frac{1}{3}$, $b = 1$, $c = 1$, and $d = -4$

Examples

- Step 2: Use the values of the parameters to describe the transformations of the Absolute Value parent function f(x) that are necessary to produce g(x).
- a = $\frac{1}{3}$; so |a| > 1, then the y-values are multiplied by a factor of $\frac{1}{3}$ to vertically compress the graph
- b = 1; there is no affect to the graph
- c = 1, so *c* < 0, then the graph will translate $|1|$ = 1 to the right
- d = -4, so *d* < 0, then the graph will translate $|4|$ units down

Examples

- Identify the key attributes of $f(x) = \frac{5}{4}|x + 2| - 1$, including domain, range, vertex, x- and y-intercepts. Write the domain and range as intervals and in set builder notation. Determine whether the vertex is a maximum or a minimum value of the function.

Examples

- Step 1: Determine the domain and range of $f(x) = \frac{5}{4}|x + 2| - 1$. The domain is always *all real numbers*
  - $(-\infty, \infty)$
  - $\{x|x \in \mathbb{R}\}$

- Since $a > 0$, the graph will open up. So the range will be numbers $f(x) > -1$
  - $(-1, \infty)$
  - $\{f(x) \mid f(x) \geq -1)$

Examples

- Step 2: Determine the vertex of the graph.
  - The vertex is $(\frac{c}{b}, d)$
    - $(\frac{-2}{1}$, -1) = (-2, -1)
  - Since a > 0, this value is a minimum

Examples

- Step 3: Determine the x-intercepts.
  - The x-intercepts are located at $(\frac{c \pm \frac{d}{a}}{b}, 0)$
    - $(\frac{-2 \pm \frac{-1}{1.25}}{1}, 0)$
    - $(\frac{-2 \pm \frac{4}{5}}{1}, 0)$
    - $(-2 \pm \frac{4}{5}, 0)$
    - $(-2 + \frac{4}{5}, 0); (-2 - \frac{4}{5}, 0)$
    - $(-2 + \frac{4}{5}, 0); (-2 - \frac{4}{5}, 0)$
    - (-1.2, 0); (-2.8, 0)

Examples

- Step 4: Determine the y-intercepts
  - The y-intercept occurs where x = 0
    - $f(x) = \frac{5}{4}|x + 2| - 1$
    - $f(x) = \frac{5}{4}|0 + 2| - 1$
    - $f(x) = \frac{5}{4}|2| - 1$
    - $f(x) = \frac{10}{4} - 1$
    - $f(x) = \frac{6}{4} = 1.5$
    - (0, 1.5)