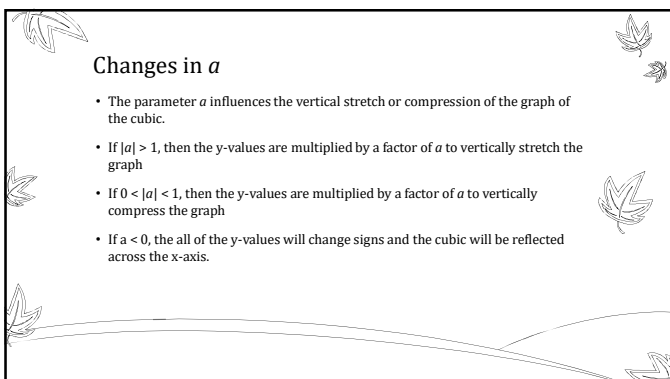


Cubic Functions

- For a cubic function, the general form is $f(x) = a(bx - c)^3 + d$, where a , b , c , and d are real numbers.
- The cubic parent function is $f(x) = x^3$
- The full family of cubic functions is generated by applying transformations to the cubic parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship



Changes in a

- The parameter a influences the vertical stretch or compression of the graph of the cubic.
- If $|a| > 1$, then the y -values are multiplied by a factor of a to vertically stretch the graph
- If $0 < |a| < 1$, then the y -values are multiplied by a factor of a to vertically compress the graph
- If $a < 0$, the all of the y -values will change signs and the cubic will be reflected across the x -axis.

Changes in b

- The parameter b influences the horizontal stretch or compression of the graph of the cubic.
- If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph
- If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph

Changes in b

- If $b < 0$, the all of the x-values will change signs and the cubic will be reflected across the y-axis.
- Since the cubic has a vertical axis of symmetry, the horizontal reflection is not noticeable in the graph.

Changes in c

- The parameter c , like b , influences the horizontal translation of the graph of the cubic.
- Note that in the general form, the sign in front of the c is negative. This means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $|\frac{c}{b}|$ to the right.
- If $c < 0$, then the graph will translate $|\frac{c}{b}|$ to the left.

Changes in d

- The parameter d influences the vertical translation of the graph of the cubic.
- If $d > 0$, then the graph of the cubic will translate $|d|$ units up.
- If $d < 0$, then the graph of the cubic will translate $|d|$ units down.

Domain and Range

- A cubic function involves cubing a number. Since every real number can be cubed, there are no domain restrictions. Therefore, the domain will always be *all real numbers*, or

$$\bullet \{x \mid x \in \mathbb{R}\}$$

- The range of a cubic function comes from a set of cubed numbers. When you multiply three positive numbers, the product is positive. When you multiply three negative numbers, the product is negative. Therefore, the range will always be *all real numbers*, or

$$\bullet \{y \mid y \in \mathbb{R}\}$$

x- and y-intercept

- A cubic equation can have as many as three x-intercepts
- Use a graphing calculator to find your x-intercepts
- If the equation of the cubic is in the general form, $y = a(bx - c)^3 + d$ then we find the y-intercept by substituting $x = 0$:
 - the y-intercept becomes $(0, -ac^3 + d)$

Maximum and Minimum values

Because the range of any cubic function is *all real numbers*, there is not an absolute maximum or minimum value for a cubic function. However, for some cubic functions, there are local maximum or minimum values.

Examples

- What transformations of the cubic parent function, $f(x) = x^3$, will result in the graph of the cubic function $g(x) = \frac{1}{3}(-6x - 2)^3 + 1$?

Examples

- Step 1: Rewrite the equation of $g(x)$ in general form to determine the values of the parameters a , b , c , and d .
 - $g(x) = a(bx - c)^3 + d$
 - $g(x) = \frac{1}{3}(-6x - 2)^3 + 1$
 - So, $a = \frac{1}{3}$, $b = -6$, $c = 2$, and $d = 1$

Examples

- Step 2: Use the values of the parameters to describe the transformations of the Cubic parent function $f(x)$ that are necessary to produce $g(x)$.
- $a = \frac{1}{3}$, so $|a| > 1$, then the y-values are multiplied by a factor of $\frac{1}{3}$ to vertically compress the graph
- $b = -6$; so $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{6}$ to horizontally compress the graph. Also, since $b < 0$, the graph will be reflected over the y-axis
- $c = 2$, so $c > 0$, then the graph will translate $\left[\frac{2}{6}\right] = \frac{1}{3}$ to the right
- $d = 1$, so $d > 0$, then the graph of the cubic will translate 1 units up

Examples

- Identify the key attributes of $f(x) = \frac{3}{4}(0.2x + 5)^3 - 1$, including domain, range, vertex, x- and y-intercepts. Write the domain and range as intervals and as inequalities.

Examples

- Step 1: Determine the domain and range of $f(x) = \frac{3}{4}(0.2x + 5)^3 - 1$
- The domain is always *all real numbers*
 - $(-\infty, \infty)$
 - $-\infty < x < \infty$
- The range is always *all real numbers*
 - $(-\infty, \infty)$
 - $-\infty < y < \infty$

Examples

- Step 2: Determine the x-intercepts.
 - Use your calculator to find your x-intercepts
 - $(-19.5, 0)$
- Step 3: Determine the y-intercepts.
 - $(0, -ac^3 + d)$
 - $(0, \frac{3}{4} + (-5)^3 - 1)$
 - $(0, 92.75)$

Examples

- Identify and compare the x-intercepts of $f(x) = x - 1$, $g(x) = (x + 3)^2$, and $h(x) = (x - 1)(x + 3)^2$.

Examples

- Step 1: Determine the x-intercepts of $f(x)$.
 - Since $f(x)$ is linear, it has one x-intercept at $(\frac{ac-d}{ab}, 0)$
 - $(\frac{1+0-(-1)}{1 \cdot 1}, 0) = (1, 0)$

Examples

- Step 2: Determine the x-intercepts of $g(x)$.

- Since $f(x)$ is a quadratic function, it will have only one x-intercept at $(\frac{-c \pm \sqrt{c^2 - 4ad}}{2a}, 0)$

$$\bullet \left(\frac{-3 \pm \sqrt{(-3)^2}}{1}, 0 \right)$$

$$\bullet \left(\frac{-3 \pm 0}{1}, 0 \right)$$

$$\bullet (-3, 0)$$

Examples

- Step 3: Determine the x-intercepts of $h(x)$.

- Using a graphing calculator, the x-intercepts are found at $(-3, 0)$ and $(1, 0)$

Examples

- Step 4: Compare the x-intercepts of $f(x)$, $g(x)$ and $h(x)$.

- $f(x)$ has only one x-intercept

- $g(x)$ has only one x-intercept

- One of the intercepts of $h(x)$ is the same as $f(x)$, and the other is the same as $g(x)$.
