2.3 Transforming and Analyzing Cubic Functions

FOCUSING QUESTION How does changing the parameters of a cubic function affect its key attributes?

LEARNING OUTCOMES

- I can compare and contrast the key attributes of a cubic function with linear and quadratic functions when I have the function as a table, graph, or written symbolically.
- I can represent the domain and range of a cubic function in a variety of ways, including interval notation, inequalities, and set builder notation.
- I can use multiple representations, including symbols, graphs, tables, and language to communicate mathematical ideas.

ENGAGE

Isabella graphed three cubic functions: $y = (x - 5)^3$, $y = (x + 7)^3$, and $y = x^3$. Which of the three graphs will have an x-intercept farthest to the right? Use mathematically appropriate vocabulary to explain your answer.

See margin.

EXPLORE

Cubic functions can be written in the general polynomial form, $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers and $a \neq 0$. As with linear and quadratic functions, some cubic functions can be written in factored form, $f(x) = a(bx - c)^3 + d$, where a, b, c, and d represent real numbers. In this factored form, the parameters a, b, c, and d transform the graph of the cubic function.

Use your graphing calculator to graph the four functions shown in each box on the same screen. Graph the first function, $Y1$, in bold or a different color. Use the graphs and tables of values on the graphing calculator to answer the questions next to the box.

INVESTIGATING a

- $Y1 = (x - 3)^3$
- $Y2 = 2(x - 3)^3$
- $Y3 = 4(x - 3)^3$
- $Y4 = 5(x - 3)^3$

1. What happens to the graph of $y = a(x - 3)^3$ when the value of a increases?
See margin

TEKS

AR.3A Compare and contrast the key attributes, including domain, range, maxima, minima, and intercepts, of a set of functions such as a set comprised of a linear, a quadratic, and an exponential function or a set comprised of an absolute value, a quadratic, and a square root function tabularly, graphically, and symbolically.

AR.7A Represent domain and range of a function using interval notation, inequalities, and set (builder) notation.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

1C Use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary.

VOCABULARY

cubic function, translation, reflection, dilation, maximum, minimum, domain, range, y-intercept, x-intercept, inflection point

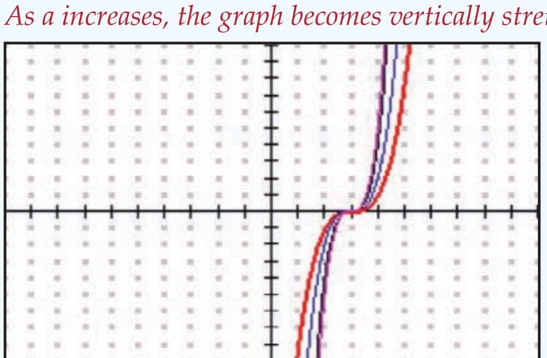
MATERIALS

- graphing calculator

ENGAGE ANSWER:

$y = (x - 5)^3$ because the constant, -5, indicates that the graph will be translated 5 units to the right. $(x + 7)^3$ is translated 7 units to the left and $y = x^3$ is the cubic parent function which has a y-intercept of zero.

1. *As a increases, the graph becomes vertically stretched because the y-values are moved farther from the x-axis.*



X	Y1	Y2	Y3	Y4
0	-27	-54	-108	-135
1	-8	-16	-32	-40
2	-1	-2	-4	-5
3	0	0	0	0
4	1	2	4	5
5	8	16	32	40
6	27	54	108	135
7	64	128	256	320
8	125	250	500	625
9	216	432	864	1080
10	343	686	1372	1715

INTEGRATE TECHNOLOGY

Encourage students to use a graphing calculator to investigate their response. Help students use what they know about linear and quadratic functions to explain their answers, using the graphs on their calculators as support.

3-8. *See page 168-A.*

STRATEGIES FOR SUCCESS

For cubic functions, changes in b affect the graph differently than changes in a. Both parameters are a multiplicative change affecting a dilation or a reflection. However, dilations with a vertically stretch or compress the graph by a factor of a with respect to the inflection point (which is (0, 0) for the parent cubic function). Dilations with b horizontally stretch or compress the graph by a factor of $\frac{1}{b}$ with respect to the y-axis regardless of where the inflection point is located.

Have students use a graphic organizer such as a Venn diagram to compare and contrast the effects of changes in a with the effects of changes in b.

- $Y1 = (x - 3)^3$
- $Y2 = 0.5(x - 3)^3$
- $Y3 = 0.25(x - 3)^3$
- $Y4 = 0.1(x - 3)^3$

2. What happens to the graph of $y = a(x - 3)^3$ when the value of a is between 0 and 1?
See margin.

- $Y1 = (x - 3)^3$
- $Y2 = -(x - 3)^3$
- $Y3 = 3(x - 3)^3$
- $Y4 = -3(x - 3)^3$

3. What happens to the graph of $y = a(x - 3)^3$ when the value of a changes signs from positive to negative?
See margin.

INVESTIGATING b

- $Y1 = (x - 6)^3$
- $Y2 = (2x - 6)^3$
- $Y3 = (3x - 6)^3$
- $Y4 = (6x - 6)^3$

4. What happens to the graph of $y = (bx - 6)^3$ when the value of b increases?
See margin.

- $Y1 = (x - 1)^3$
- $Y2 = (0.5x - 1)^3$
- $Y3 = (0.25x - 1)^3$
- $Y4 = (0.1x - 1)^3$

5. What happens to the graph of $y = (bx - 1)^3$ when the value of b is between 0 and 1?
See margin.

- $Y1 = (x - 1)^3$
- $Y2 = (-x - 1)^3$
- $Y3 = (0.5x - 1)^3$
- $Y4 = (-0.5x - 1)^3$

6. What happens to the graph of $y = (bx - 1)^3$ when the value of b changes signs from positive to negative?
See margin.

INVESTIGATING c AND d

7. You have investigated the parameters c and d with linear and quadratic functions. Make a conjecture about how these parameters will affect the graphs of cubic functions.
See margin.

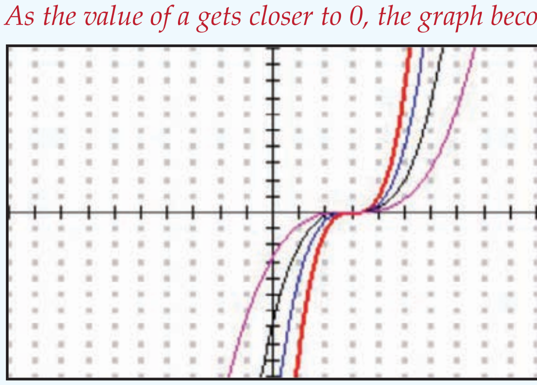
Graph the functions, generate a table of values, and use the graphs and table of values below test your hypothesis.

- $Y1 = x^3$
- $Y2 = (x - 3)^3$
- $Y3 = (x - 5)^3$
- $Y4 = (x + 2)^3$
- $Y5 = (x + 4)^3$

- $Y1 = (x - 2)^3$
- $Y2 = (x - 2)^3 + 2$
- $Y3 = (x - 2)^3 + 5$
- $Y4 = (x - 2)^3 - 3$
- $Y5 = (x - 2)^3 - 6$

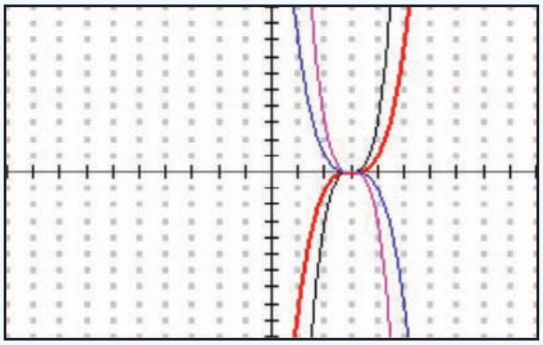
8. How does your conjecture compare the results of the graphs?
See margin.

2. *As the value of a gets closer to 0, the graph becomes more vertically compressed because the y-values are moved closer to the x-axis.*



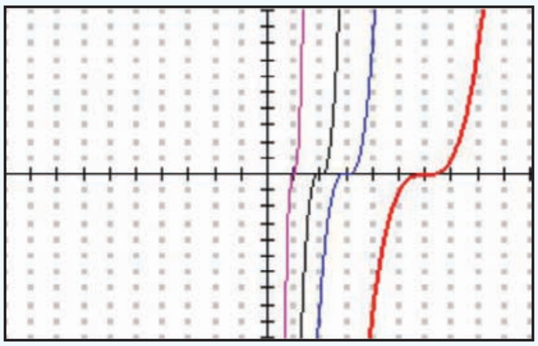
X	Y1	Y2	Y3	Y4
0	-27	-13.5	-6.75	-2.7
1	-8	-4	-2	-.8
2	-1	-.5	-.25	-.1
3	0	0	0	0
4	1	.5	.25	.1
5	8	4	2	.8
6	27	13.5	6.75	2.7
7	64	32	16	6.4
8	125	62.5	31.25	12.5
9	216	108	54	21.6
10	343	171.5	85.75	34.3

3. *When a changes signs, the graph is reflected across the x-axis.*



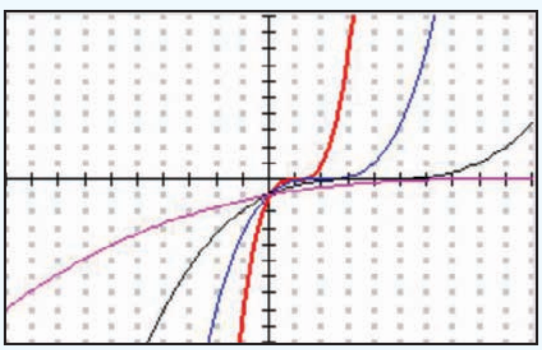
X	Y1	Y2	Y3	Y4
0	-27	27	-81	81
1	-8	8	-24	24
2	-1	1	-3	3
3	0	0	0	0
4	1	-1	3	-3
5	8	-8	24	-24
6	27	-27	81	-81
7	64	-64	192	-192
8	125	-125	375	-375
9	216	-216	648	-648
10	343	-343	1029	-1029

4. *As b increases, the graph becomes more horizontally compressed by a factor of $\frac{1}{b}$ because the points with the same y-value are moved closer to the y-axis.*



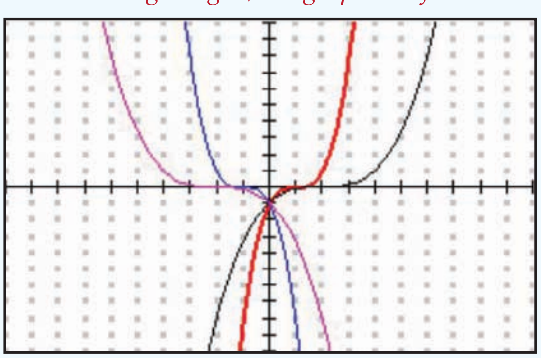
0	-216	-216	-216	-216
1	-125	-64	-27	0
2	-64	-8	0	216
3	-27	0	27	1728
4	-8	8	216	5832
5	-1	64	729	13824
6	0	216	1728	27000
7	1	512	3375	46656
8	8	1000	5832	74088
9	27	1728	9261	110592
10	64	2744	13824	157464

5. *As the value of b gets closer to 0, the graph becomes more horizontally stretched by a factor of $\frac{1}{b}$ because the points with the same y-values are moved farther from the y-axis.*



X	Y1	Y2	Y3	Y4
0	-1	-1	-1	-1
1	0	-.125	-.4219	-.729
2	1	0	-.125	-.512
3	8	.125	-.0156	-.343
4	27	1	0	-.216
5	64	3.375	.01563	-.125
6	125	8	.125	-.064
7	216	15.625	.42188	-.027
8	343	27	1	-.008
9	512	42.875	1.9531	-.001
10	729	64	3.375	0

6. *When b changes signs, the graph is reflected across the y-axis.*



X	Y1	Y2	Y3	Y4
0	-1	-1	-1	-1
1	0	-8	-.125	-3.375
2	1	-27	0	-8
3	8	-64	.125	-15.63
4	27	-125	1	-27
5	64	-216	3.375	-42.88
6	125	-343	8	-64
7	216	-512	15.625	-91.13
8	343	-729	27	-125
9	512	-1000	42.875	-166.4
10	729	-1331	64	-216

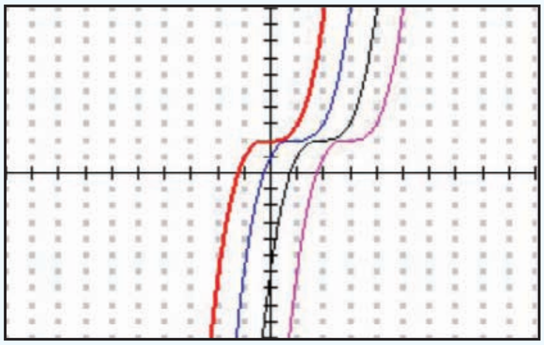
7. *Possible response: The parameter c will horizontally translate the graph of a cubic function by a factor of c units and the parameter d will vertically translate the graph of a cubic function by d units.*

8. *Possible response: The value of c translates the graph horizontally by c units. When $c < 0$, the graph is translated to the left $\frac{c}{b}$ units and when $c > 0$, the graph is translated to the right $\frac{c}{b}$ units.*

The value of d translates the graph vertically by d units. When $d < 0$, the graph is translated down d units and when $d > 0$, the graph is translated up d units.

9. *The domain and range of the cubic parent function, $y = x^3$, are both all real numbers. The parameters a, b, c, and d do not affect the domain or range of the cubic function.*

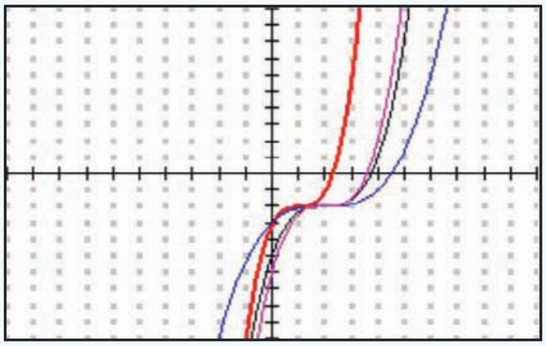
10.



X	Y1	Y2	Y3	Y4
0	2	1	-6	-25
1	3	2	1	-6
2	10	3	2	1
3	29	10	3	2
4	66	29	10	3
5	127	66	29	10
6	218	127	66	29
7	345	218	127	66
8	514	345	218	127
9	731	514	345	218
10	1002	731	514	345

x	$Y1 = x^3 + 2$	$Y2 = (x - 1)^3 + 2$	$Y3 = (x - 2)^3 + 2$	$Y4 = (x - 3)^3 + 2$
0	$0^3 + 2 = 2$	$(0 - 1)^3 + 2$ $= (-1)^3 + 2$ $= -1 + 2 = 1$	$(0 - 2)^3 + 2$ $= (-2)^3 + 2$ $= -8 + 2 = -6$	$(0 - 3)^3 + 2$ $= (-3)^3 + 2$ $= -27 + 2 = -25$

11.



X	Y1	Y2	Y3	Y4
0	-3	-3	-5	-6
1	-2	-2.125	-2.375	-2.5
2	-1	-2	-2	-2
3	6	-1.875	-1.625	-1.5
4	25	-1	1	2
5	62	1.375	8.125	11.5
6	123	6	22	30
7	214	13.625	44.875	60.5
8	341	25	79	106
9	510	40.875	126.63	169.5
10	727	62	190	254

x	$Y1 = (x - 1)^3 - 2$	$Y2 = (0.5x - 1)^3 - 2$	$Y3 = 3(0.5x - 1)^3 - 2$	$Y4 = 4(0.5x - 1)^3 - 2$
0	$(0 - 1)^3 - 2$ $= (-1)^3 - 2$ $= -1 - 2 = -3$	$(0 - 1)^3 - 2$ $= (-1)^3 - 2$ $= -1 - 2 = -3$	$3(0 - 1)^3 - 2$ $= 3(-1)^3 - 2$ $= 3(-1) - 2$ $= -3 - 2 = -5$	$4(0 - 1)^3 - 2$ $= 4(-1)^3 - 2$ $= 4(-1) - 2$ $= -4 - 2 = -6$

Use your investigations to answer the following questions.

9. How do changes in the parameters a, b, c, and d affect the domain and range of a cubic function?
See page 169-A.

10. Graph the functions following functions and use the graphs and table of values to explain how the y-intercept of each function compares to the values of c and d.

- $y = x^3 + 2$
- $y = (x - 1)^3 + 2$
- $y = (x - 2)^3 + 2$
- $y = (x - 3)^3 + 2$

The y-intercept moves vertically $(-c)^3 + d$ units. See page 169-A for graph and tables.

11. Graph the following functions and use the graphs and table of values to explain how the y-intercept of each function compares to the values of a and b.

- $y = (x - 1)^3 - 2$
- $y = (0.5x - 1)^3 - 2$
- $y = 3(0.5x - 1)^3 - 2$
- $y = 4(0.5x - 1)^3 - 2$

The value of b does not affect the y-intercept and the value of a multiplies the vertical movement generated by c. See page 169-A for graph and tables.

12. How could you write the coordinates of the y-intercept using the parameters a, b, c, and d?
The y-intercept is the point $(0, -ac^3 + d)$

REFLECT

- **The parameters *a* and *b* are multiplicative parameters. How do changes in *a* and *b* affect the graph of $y = x^3$?**
  See margin.
- **The parameters *c* and *d* are additive parameters. How do changes in *c* and *d* affect the graph of $y = x^3$?**
  See margin.
- **How do the transformations generated by *a*, *b*, *c*, and *d* for cubic functions compare to the related transformations for linear and quadratic functions?**
  See margin.
- **Which parameters affect the *y*-intercept of a cubic function? Why do you think that is the case?**
  See margin.

USING TECHNOLOGY

The x-intercepts of any function, including cubic functions, can be calculated from the graph screen of a graphing calculator. The x-coordinates of the x-intercepts of the graph of a function are the same as the zeroes of the function or the roots of the equation.

Graphing calculators have a feature that allows you to calculate the zero of the graph of a function. Students can use this feature to estimate or calculate the x-coordinates of the x-intercepts of a cubic function.

REFLECT ANSWERS:

The parameter a generates a vertical dilation or a reflection across the x-axis. If $a > 1$, then the graph is vertically stretched by a factor of a. If $0 < a < 1$, then the graph is vertically compressed by a factor of a. If $a < 0$, then the graph is reflected across the x-axis.

The parameter b generates a horizontal dilation or a reflection across the y-axis. If $b > 1$, then the graph is horizontally compressed by a factor of $\frac{1}{b}$. If $0 < b < 1$, then the graph is horizontally stretched by a factor of $\frac{1}{b}$. If $b < 0$, then the graph is reflected across the y-axis.

The parameter c works with the parameter b to generate a horizontal translation. If $c > 0$, then the graph is horizontally translated $\frac{c}{b}$ units to the right. If $c < 0$, then the graph is horizontally translated $\frac{c}{b}$ units to the left.

The parameter d generates a vertical translation. If $d > 0$, then the graph is vertically translated d units up. If $d < 0$, then the graph is vertically translated d units down.

The parameters a, b, c, and d generate the same types of transformations for linear, quadratic, and cubic functions.

The y-intercept is affected by the parameters a, c, and d. The parameters c and d influence horizontal and vertical translations which move the inflection point of the graph. The parameter a influences vertical dilations and reflections which stretch, compress, or reflect the y-intercept. The parameter b does not affect the y-intercept because horizontal dilations are performed with respect to the y-axis, which has a value of $x = 0$. Multiplying b by 0 does not change the y-coordinate of the y-intercept.

INSTRUCTIONAL HINT

Students began a graphic organizer for transforming functions in section 2.1 and continued it in section 2.2. Have students add cubic functions to this graphic organizer. By summarizing the affects of changes in *a*, *b*, *c*, and *d* for each type of function, students should notice patterns and have an easy reference sheet as they practice.

EXPLAIN

A cubic function is a relationship between an independent variable, usually x, and a dependent variable, usually y or $f(x)$, that generates an s-shaped graph. A cubic function involves generating dependent variable values by cubing the independent variable values.

Watch Explain and You Try It Videos

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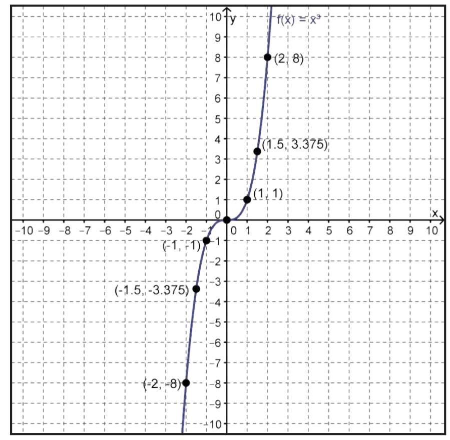
$f(x) = x^3$

x	$f(x)$
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27



The graphs of quadratic functions have a vertex, which is where the graph of the function changes from decreasing to increasing (if $a > 0$) or from increasing to decreasing (if $a < 0$). Cubic functions also have a key point, the **inflection point**. The inflection point of a cubic function is where the curvature of the graph changes from appearing to open downward to appearing to open upward. In the cubic parent function, $f(x) = x^3$, the inflection point is the origin, (0, 0).

For a **cubic function**, the factored form that can be used to transform the parent function is $f(x) = a(bx - c)^3 + d$, where a, b, c, and d are real numbers.

The full family of cubic functions is generated by applying transformations to the cubic parent function. Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship.

CHANGES IN *a*

The parameter a influences the vertical dilation of the graph.

- If $|a| > 1$, then the y-values of the original function are multiplied by a factor of a in order to vertically stretch the graph.
- If $0 < |a| < 1$, then the y-values of the original function are multiplied by a factor of a in order to vertically compress the graph.

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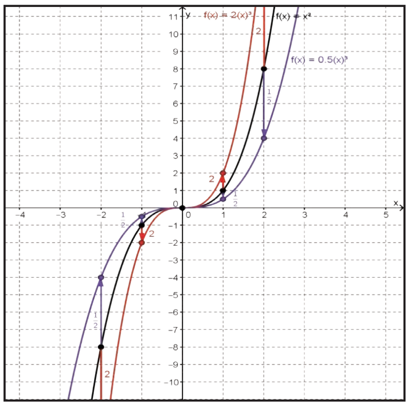
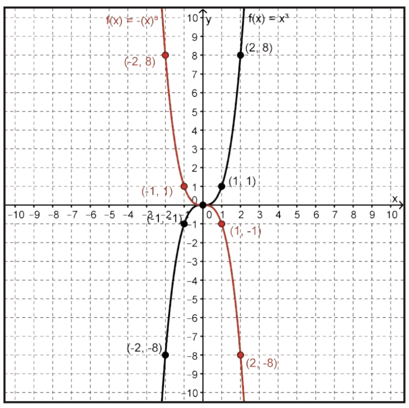


x	$f(x) = x^3$	$f(x) = 2(x)^3$	$f(x) = 0.5(x)^3$
-2	-8	-16	-4
-1	-1	-2	-0.5
0	0	0	0
1	1	2	0.5
2	8	16	4

× 2 × 0.5

The parameter a also affects the orientation of the graph. If $a < 0$, then the graph will be reflected across the x-axis.



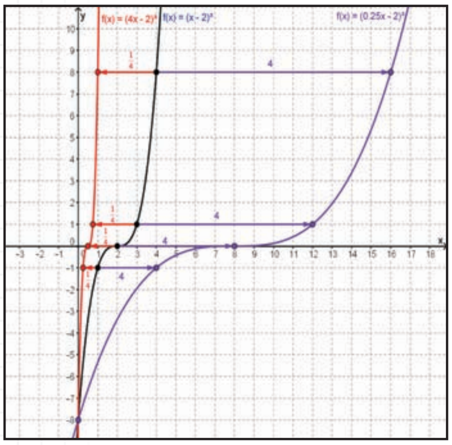
x	$f(x) = (x)^3$	$f(x) = -(x)^3$
-2	-8	8
-1	-1	1
0	0	0
1	1	-1
2	8	-8

× -1

CHANGES IN b

The parameter b influences the horizontal stretch or compression of the graph.

- If $|b| > 1$, then the x-values of the original function are multiplied by a factor of $\frac{1}{|b|}$, which will be a multiplier that is less than 1, in order to horizontally compress the graph.
- If $0 < |b| < 1$, then the x-values of the original function are multiplied by a factor of $\frac{1}{|b|}$, which will be a multiplier that is greater than 1, in order to horizontally stretch the graph.



x	$f(x) = (x-2)^3$
0	-8
1	-1
2	0
3	1
4	8

x	$f(x) = (4x-2)^3$
0	-8
$\frac{1}{4}$	-1
$\frac{1}{2}$	0
$\frac{3}{4}$	1
1	8

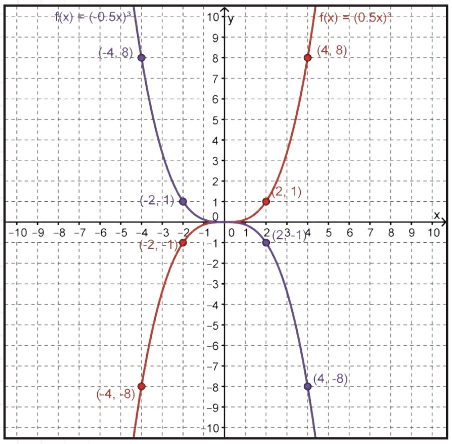
Domain (x) values are multiplied by $\frac{1}{4}$ in order to generate the same range (y) value. This multiplication results in a horizontal compression of the graph.

x	$f(x) = (x-2)^3$
0	-8
1	-1
2	0
3	1
4	8

x	$f(x) = (\frac{1}{4}x-2)^3$
0	-8
4	-1
8	0
12	1
16	8

Domain (x) values are multiplied by 4 in order to generate the same range (y) value. This multiplication results in a horizontal stretch of the graph.

The parameter b also affects the orientation of the graph. If $b < 0$, then all of the x-values will change signs and the graph will be reflected across the y-axis.



x	$f(x) = (0.5x)^3$
-4	-8
-2	-1
0	0
2	1
4	8

x	$f(x) = (-0.5x)^3$
4	-8
2	-1
0	0
-2	1
-4	8

Domain (x) values are multiplied by −1 in order to generate the same range (y) value. This multiplication moves the points from Quadrant I to Quadrant II and from Quadrant III to Quadrant IV, generating a reflection across the y-axis.

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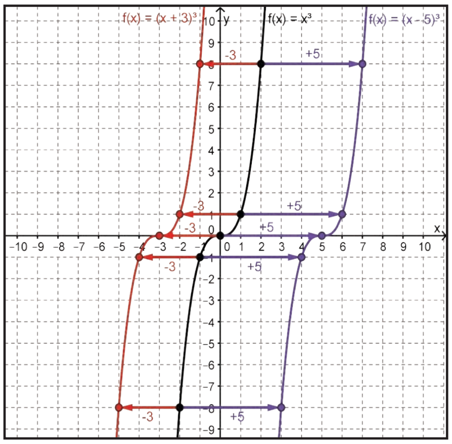
CHANGES IN c

The parameter c influences the horizontal translation of the graph. Notice that in the general form, $f(x) = a(bx - c)^3 + d$, the sign in front of c is negative. That means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation.

- If $c > 0$, then the graph of the curve will translate $|\frac{c}{b}|$ units to the right.
- If $c < 0$, then the graph of the curve will translate $|\frac{c}{b}|$ units to the left.



x	$f(x) = x^3$
-2	-8
-1	-1
0	0
1	1
2	8

x	$f(x) = (x-5)^3$
3	-8
4	-1
5	0
6	1
7	8

Domain (x) values are increased by 5 in order to generate the same range (y) value. This addition results in a horizontal translation of the graph to the right.

x	$f(x) = x^3$
-2	-8
-1	-1
0	0
1	1
2	8

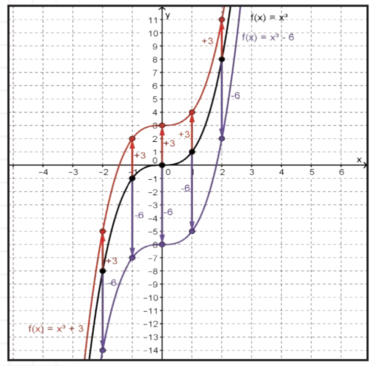
x	$f(x) = (x+3)^3$
-5	-8
-4	-1
-3	0
-2	1
-1	8

Domain (x) values are decreased by 3 in order to generate the same range (y) value. This subtraction (addition with a negative number) results in a horizontal translation of the graph to the left.

CHANGES IN d

The parameter d influences the horizontal translation of the graph.

- If $d > 0$, then the graph will translate $|d|$ units up.
- If $d < 0$, then the graph will translate $|d|$ units down.



x	$f(x) = x^3$	$f(x) = x^3 - 6$	$f(x) = x^3 + 3$
-2	-8	-14	-5
-1	-1	-7	2
0	0	-6	3
1	1	-5	4
2	8	2	11

−6 +3

Each of these parameters also has an effect on key attributes of a cubic function.

DOMAIN AND RANGE

A cubic function involves cubing a number. Every real number can be cubed, so there are no domain restrictions on cubic functions. The domain is always all real numbers, or $\{x \mid x \in \mathbb{R}\}$.

The range of a cubic function comes from a set of cubed numbers. When you multiply three positive numbers, the product is positive. When you multiply three negative numbers, the product is negative. Therefore, the range could be any of the set of real numbers, or $\{y \mid y \in \mathbb{R}\}$.

The domain and range of a cubic function, all real numbers, can be represented using interval notation, set builder notation or inequalities.

	DOMAIN	RANGE
INTERVAL NOTATION	$(-\infty, \infty)$	$(-\infty, \infty)$
SET BUILDER NOTATION	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$
INEQUALITIES	$-\infty < x < \infty$	$-\infty < y < \infty$

y-INTERCEPT

The y-intercept of any function is the point $(0, y)$, or the point where the graph intersects the y-axis. For a cubic function, you can calculate the y-intercept by substituting $x = 0$ into the factored form, $y = a(bx - c)^3 + d$.

$$y = a(b(0) - c)^3 + d$$
$$y = a(0 - c)^3 + d$$
$$y = a(-c)^3 + d$$
$$y = -ac^3 + d$$

The parameters a, c, and d each influence the y-intercept of a cubic function. The parameter d vertically shifts the entire graph, including the y-intercept, and the parameter c horizontally shifts the entire graph, including the y-intercept. The parameter a vertically dilates or reflects the entire graph, including the y-intercept. Each of these motions affects where the graph intersects the y-axis.

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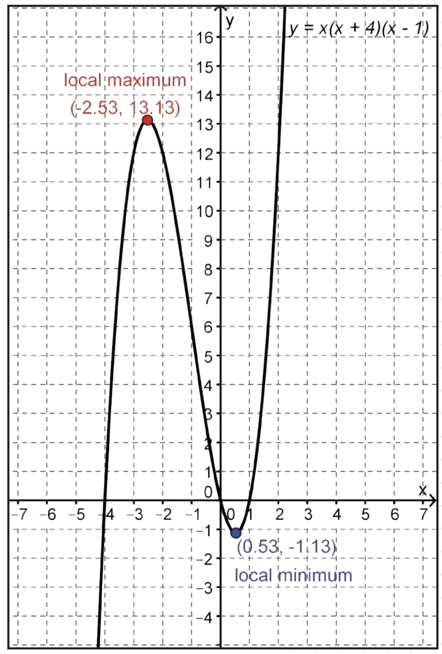
MAXIMUM AND MINIMUM VALUES

Because the range of any cubic function is all real numbers, there is not an absolute maximum value or an absolute minimum value for a cubic function. However, for some cubic functions, there are local maximum or minimum values.

For example, the graph of $f(x) = x(x + 4)(x - 1)$ is shown. In this cubic function, as x increases from the left, the y-values reach a relative, or local maximum when $x \approx -2.53$. As x continues to increase, the y-values reach a relative, or local minimum when $x \approx 0.53$. Local maximum and minimum values are useful for solving problems when only a portion of the cubic function models the situation.

You can calculate the coordinates of local maximum and local minimum values using graphing technology.



KEY ATTRIBUTES OF CUBIC FUNCTIONS

A cubic function has several important key attributes:

- The domain of a cubic function is all real numbers.
- The range of a cubic function is all real numbers.
- There is neither an absolute maximum value nor an absolute minimum value for a cubic function.
- Cubic functions may have a local maximum value or a local minimum value.
- A cubic function has one y-intercept at $(0, -ac^3 + d)$.
- A cubic function may have up to three x-intercepts.
- The local maximum value, local minimum value, and x-intercepts of a cubic function can be calculated using appropriate graphing technology.

EXAMPLE 1

What transformations of the cubic parent function, $f(x) = x^3$, will result in the graph of the cubic function $g(x) = \frac{1}{3}(-6x - 2)^3 + 1$?

STEP 1 **Determine the values of the parameters *a*, *b*, *c* and *d*.**

$g(x) = \frac{1}{3}(-6x - 2)^3 + 1$

Therefore, $a = \frac{1}{3}$, $b = -6$, $c = 2$, and $d = 1$.

STEP 2 **Use the values of the parameters to describe the transformations of the cubic parent function *f(x)* that are necessary to produce *g(x)*.**

$a = \frac{1}{3}$, so $|a| < 1$. The range values (y-coordinates) of the cubic parent function are multiplied by a factor of $\frac{1}{3}$ in order to vertically compress the graph of the function.

$b = -6$, so $|b| > 1$. The domain values (x-coordinates) of the parent cubic function are multiplied by a factor of $\frac{1}{|-6|} = \frac{1}{6}$, which will be a multiplier that is less than 1, in order to horizontally compress the graph. Also, since $b < 0$, the graph will be reflected over the y-axis.

$c = 2$, so $c > 0$. The graph of the cubic parent function will translate $|\frac{2}{-6}| = \frac{1}{3}$ units to the left.

$d = 1$, so $d > 0$. The graph of the cubic parent function will translate $|1| = 1$ unit up.

The graph of g(x) is produced by transforming the cubic parent function f(x) by vertically compressing its graph by a factor of one third, horizontally compressing its graph by a factor of one sixth, reflecting its graph across the y-axis, and translating its graph one-third unit to the right and one unit up.

YOU TRY IT! #1

What transformations of the cubic parent function, $y = x^3$, will result in the graph of the cubic function $y = -2(x + 3)^3 + 4.5$?

See margin.

ADDITIONAL EXAMPLES

What transformations of the cubic parent function, $f(x) = x3$, will result in the graph of the cubic functions below?

1. $g(x) = 0.6(-4x - 5)^3 - 7.3$

The graph of g(x) is produced by vertically compressing the graph of f(x) by a factor of 0.6, horizontally compressing its graph by a factor of 0.25, reflecting its graph across the y-axis, and translating its graph 1.25 units to the left and 7.3 units down.

2. $h(x) = -10(\frac{1}{2}x - 8)^3 + 12$

The graph of h(x) is produced by vertically stretching the graph of f(x) by a factor of 10, horizontally stretching its graph by a factor of 2, reflecting its graph over the x-axis, and translating its graph 16 units to the right and 12 units up.

QUESTIONING STRATEGY

In *ADDITIONAL EXAMPLE #1* $c > 0$. This should cause a translation to the right, however the function translates to the left. What causes this unexpected change?

YOU TRY IT! #1 ANSWER:

The graph of $y = -2(x + 3)^3 + 4.5$ is produced by transforming the cubic parent function by vertically stretching its graph by a factor of two, reflecting its graph over the x-axis, and translating its graph three units to the left and four and a half units up.

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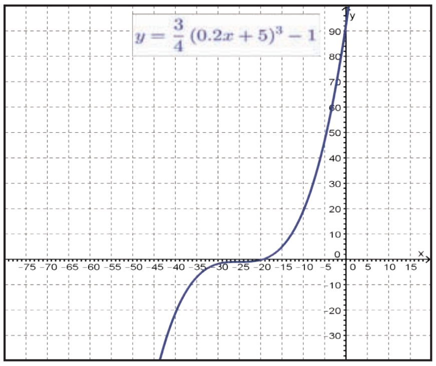
EXAMPLE 2

Identify the key attributes of $y = \frac{3}{4}(0.2x + 5)^3 - 1$, including domain, range, x-intercept(s), and y-intercept. Write the domain and range as intervals and as inequalities.



STEP 1 **Determine the domain and range of $y = \frac{3}{4}(0.2x + 5)^3 - 1$.**

Since the function is cubic, its domain and range are all real numbers. This is confirmed by the graph. The domain contains all real values of x. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written as an inequality is $-\infty < x < \infty$.

The range of this cubic function contains all real values of y. As an interval, this is written as $(-\infty, \infty)$. The same information about the range written as an inequality is $-\infty < y < \infty$.

STEP 2 **Determine the x-intercept(s) of the cubic function $y = \frac{3}{4}(0.2x + 5)^3 - 1$.**

According to the provided graph, this cubic function has only one x-intercept. Using graphing technology, the x-intercept is located at $(-19.5, 0)$.

This is confirmed by the provided graph that shows the curve crosses the x-axis at a value of x that is approximately -20.

STEP 3 **Determine the y-intercept of $y = \frac{3}{4}(0.2x + 5)^3 - 1$.**

y-intercepts of functions occur where the domain or input value $x = 0$.

$y = \frac{3}{4}(0.2x + 5)^3 - 1$

$y = \frac{3}{4}(0.2(0) + 5)^3 - 1 = \frac{3}{4}(5)^3 - 1 = \frac{3}{4}(125) - 1 = 93.75 - 1 = 92.75.$

The y-intercept of the quadratic function is $(0, 92.75)$. The graph confirms this since the curve intersects the y-axis at a point between $(0, 80)$ and $(0, 100)$.

The domain of $y = \frac{3}{4}(0.2x + 5)^3 - 1$ is $(-\infty, \infty)$ or $-\infty < x < \infty$. The range is $(-\infty, \infty)$ which may also be written as $-\infty < y < \infty$. This cubic function has an x-intercept at $(-19.5, 0)$. The y-intercept of $y = \frac{3}{4}(0.2x + 5)^3 - 1$ is $(0, 92.75)$.

ADDITIONAL EXAMPLES

Identify the key attributes of the functions below including domain, range, x-intercept(s), and y-intercept. Write the domain and range as intervals and inequalities.

1. $y = 7(-3x - 12)^3 - 14$

Domain: $(-\infty, \infty)$ or $-\infty < x < \infty$
Range: $(-\infty, \infty)$ or $-\infty < y < \infty$.
x-intercept $(-4.42, 0)$,
y-intercept $(0, -12110)$

2. $y = -\frac{2}{3}(x + 9)^3 + 4$

Domain: $(-\infty, \infty)$ or $-\infty < x < \infty$
Range: $(-\infty, \infty)$ or $-\infty < y < \infty$.
x-intercept $(-7.18, 0)$,
y-intercept $(0, -482)$

ADDITIONAL EXAMPLE

Identify the key attributes of the cubic function $g(x) = (x - 4)(x + 2)(3x + 1)$, including its domain, range, relative minimum, relative maximum, x-intercept(s), and y-intercept. Write the domain and range of $g(x)$ as inequalities and in set builder notation. Use graphing technology to determine the relative minimum, relative maximum, and x-intercepts.

Domain: $-\infty < x < \infty$ or $\{x \mid \in \mathbb{R}\}$
Range: $(-\infty, \infty)$ or $\{g(x) \mid (x) \in \mathbb{R}\}$
Relative minimum at $(2.34, -57.8)$
Relative maximum at $(-1.23, 10.8)$
x-intercepts $(4, 0)$, $(-2, 0)$, and $(-\frac{1}{3}, 0)$.
y-intercept $(0, -8)$

YOU TRY IT! #2 ANSWER:

The domain of $f(x)$ is $-\infty < x < \infty$, which can also be written as $\{x \mid x \in \mathbb{R}\}$. The range of $f(x)$ is $-\infty < f(x) < \infty$ or $\{f(x) \mid f(x) \in \mathbb{R}\}$. The cubic function $f(x)$ has a relative minimum at $(2, -9)$ and a relative maximum at $(-0.33, 3.7)$. The x-intercepts are $(0.5, 0)$, $(3, 0)$ and $(-1, 0)$. The y-intercept is $(0, 3)$.

ADDITIONAL EXAMPLES

Identify and compare the x-intercepts of the following functions.

1.

$f(x) = 2(x + 6) - 8$

$g(x) = 2(x + 6)^2 - 8$

$h(x) = 2(x + 6)^3 - 8$

The x-intercept of $f(x)$ is (-2, 0). The x-intercepts of $g(x)$ are (-4, 0) and (-8, 0). The x-intercept of $h(x)$ is (-4.42, 0).

2.

$f(x) = -\frac{1}{2}(4x - 2) - 1$

$g(x) = -\frac{1}{2}(4x - 2)^2 - 1$

$h(x) = -\frac{1}{2}(4x - 2)^3 - 1$

The x-intercept of $f(x)$ is (0, 0). $g(x)$ does not have any x-intercepts. The x-intercept of $h(x)$ is (0.19, 0).

YOU TRY IT! #2

Identify the key attributes of the cubic function $f(x) = (x + 1)(x - 3)(2x - 1)$, including its domain, range, relative minimum, relative maximum, x-intercept(s), and y-intercepts. Write the domain and range of $f(x)$ as inequalities and in set builder notation. Use graphing technology to determine the relative minimum, relative maximum, and x-intercepts.

x	$f(x)$
− 3	− 84
− 2	− 25
− 1	0
0	3
1	− 4
2	− 9
3	0

See margin.

EXAMPLE 3

Identify and compare the x-intercepts of $f(x) = x - 1$, $g(x) = (x + 3)^2$ and $h(x) = (x - 1)(x + 3)^2$.

STEP 1 **Determine the x-intercept(s) of $f(x)$.**

Since $f(x)$ is a linear function, $f(x)$ has one x-intercept at $\left(\frac{ac - d}{ab}, 0\right)$.

$$\frac{ac - d}{ab} = \frac{(1)(0) - (-1)}{(1)(1)} = \frac{1}{1} = 1$$

The x-intercept of $f(x)$ is (1, 0).

STEP 2 **Determine the x-intercept(s) of $g(x)$.**

Since $g(x)$ is a quadratic function, the x-intercept(s) are located at $\left(\frac{c \pm \sqrt{\frac{-d}{a}}}{b}, 0\right)$.

$$\frac{c \pm \sqrt{\frac{-d}{a}}}{b} = \frac{-3 \pm \sqrt{\frac{-0}{1}}}{1} = \frac{-3 \pm \sqrt{0}}{1} = \frac{-3 \pm 0}{1} = -3$$

The x-intercept of $g(x)$ is $(-3, 0)$.

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STEP 3 **Determine the x-intercept(s) of $h(x)$.**

Using graphing technology, the x-intercepts of $h(x)$ are $(-3, 0)$ and $(1, 0)$.

STEP 4 **Compare the x-intercepts of $f(x)$, $g(x)$, and $h(x)$.**

The function $f(x)$ has only one x-intercept. The function $g(x)$ also has one x-intercept. One of the intercepts of $h(x)$ is the same as the x-intercept of $f(x)$ and the other is the same as the x-intercept of $g(x)$.

The x-intercept of f(x) is (1, 0). The x-intercept of g(x) is (– 3, 0). The x-intercepts of h(x) are (– 3, 0) and (1, 0). Since f(x) is a linear function, it has only one x-intercept. The quadratic function g(x) has only one x-intercept because the graph touches but does not cross the x-axis. One of the intercepts of h(x) is the same as the x-intercept of f(x) and the other is the same as the x-intercept of g(x). The cubic function h(x) has only two x-intercepts because it has a relative maximum on the x-axis.

YOU TRY IT! #3

Identify and compare the x- and y-intercepts of the following functions:

- $f(x) = -(x-5)+4$
- $g(x) = -(x-5)^2+4$
- $h(x) = -(x-5)^3+4$

See margin.

EXAMPLE 4

Identify and compare the domains and ranges of the following functions:

- $f(x) = -2(x+7)^2-3$
- $g(x) = 2(x+3)-7$
- $h(x) = -3(x+2)^3-7$

Write the domain and range of each function as inequalities, as intervals, and in set builder notation.

YOU TRY IT! #3 ANSWER:

The x-intercept of f(x) is (9, 0). The x-intercepts of g(x) are (3, 0) and (7, 0). The x-intercept of h(x) is (6.59, 0). The cubic function h(x) has only one x-intercept instead because its inflection point is above the x-axis. Since f(x) is a linear function, it has just one x-intercept. Although all three functions appear to have similar equations, their x-intercepts are different because the parameters a, b, c, and d affect the x-intercepts of linear, quadratic, and cubic functions differently.

The y-intercept of f(x) is (0, 9). The y-intercept of g(x) is (0, – 21). The y-intercept of h(x) is (0, 129). Any function can have only one y-intercept. Although the equations that describe the functions are very similar, their y-intercepts are different because the parameters a, b, c, and d also affect the y-intercepts of linear, quadratic, and cubic functions differently.

ADDITIONAL EXAMPLE

Identify and compare the domains and ranges of the following functions:

$p(x) = 3(x + 5) - 6$

$q(x) = 3(x + 5)^2 - 6$

$r(x) = 3(x + 5)^3 - 6$

Write the domain and range of each function as inequalities, as intervals, and in set builder notation.

The domains of all three of these functions contain all real numbers.

- *Inequality:* $-\infty < x < \infty$
- *Interval:* $(-\infty, \infty)$
- *Set Builder:* $\{x \mid x \in \mathbb{R}\}$

The ranges of p(x) and r(x) contain all real numbers.

- *Inequality:* $-\infty < p(x) < \infty$
- *Interval:* $(-\infty, \infty)$
- *Set Builder:* $\{p(x) \mid p(x) \in \mathbb{R}\}$
- *Inequality:* $-\infty < r(x) < \infty$
- *Interval:* $(-\infty, \infty)$
- *Set Builder:* $\{r(x) \mid r(x) \in \mathbb{R}\}$

The range of q(x) is limited by values greater than or equal to -6.

- *Inequality:* $-6 \le q(x) < \infty$
- *Interval:* $[-6, \infty)$
- *Set Builder:* $\{q(x) \mid q(x) \ge -6\}$

STEP 1 **Determine the domain and range of $f(x)$.**

Since $f(x)$ is a quadratic function, the domain contains all real values of x. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written in set builder notation is $\{x \mid x \in \mathbb{R}\}$. Written as an inequality, $-\infty < x < \infty$.

Using the equation, the value of a indicates that the parabola opens downward. The only other parameter that affects the range of a quadratic function is d. Since $d = -3$, the range of this quadratic function contains all real values of $f(x)$ that are less than or equal to negative three. As an interval, this is written as $(-\infty, -3]$. The same information about the range written in set builder notation is $\{f(x) \mid f(x) \le -3\}$. As an inequality, this is written $-\infty < f(x) \le -3$.

STEP 2 **Determine the domain and range of $g(x)$.**

Since $g(x)$ is a linear function, its domain contains all real values of x. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written in set builder notation is $\{x \mid x \in \mathbb{R}\}$. Written as an inequality, $-\infty < x < \infty$.

Since $g(x)$ is a linear function, its range contains all real values of $g(x)$. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written in set builder notation is $\{g(x) \mid g(x) \in \mathbb{R}\}$. Written as an inequality, $-\infty < g(x) < \infty$.

STEP 3 **Determine the domain and range of $h(x)$.**

Since $h(x)$ is a cubic function, its domain contains all real values of x. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written in set builder notation is $\{x \mid x \in \mathbb{R}\}$. Written as an inequality, $-\infty < x < \infty$.

Since $h(x)$ is a cubic function, its range contains all real values of $h(x)$. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written in set builder notation is $\{h(x) \mid h(x) \in \mathbb{R}\}$. Written as an inequality, $-\infty < h(x) < \infty$.

STEP 4 **Compare the domains of $f(x)$, $g(x)$ and $h(x)$ and the ranges of $f(x)$, $g(x)$ and $h(x)$.**

The domains of all three of these functions are the same because the functions are polynomials. The ranges of $g(x)$ and $h(x)$ contain all real numbers since $g(x)$ is a linear function and $h(x)$ is a cubic function. The range of the quadratic function $f(x)$ does not contain all real numbers but is restricted to only those real numbers less than or equal to negative three.

YOU TRY IT! #4 ANSWER:

The domains of all three of these functions contain all real numbers because all the functions are polynomials, f(x) having degree three, g(x) having degree one, and h(x) having degree two. The range of f(x) contains all real numbers since f(x) is a cubic function. The range of g(x) contains all real numbers since g(x) is a linear function. The range of the quadratic function h(x) does not contain all real numbers but is restricted to only those real numbers greater than or equal to the value of its parameter d. The reason the range of g(x) contains values greater than or equal to five rather than less than or equal to five is that the value of its parameter a is positive.

	NOTATION	$f(x) = \frac{1}{2}(x-4)^3 + 5$	$g(x) = \frac{1}{2}(x-4) + 5$	$h(x) = \frac{1}{2}(x-4)^2 + 5$
DOMAIN	INEQUALITY	$-\infty < x < \infty$		
	INTERVAL	$(-\infty, \infty)$		
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$		
RANGE	INEQUALITY	$-\infty < f(x) < \infty$	$-\infty < g(x) < \infty$	$5 \le h(x) < \infty$
	INTERVAL	$(-\infty, \infty)$	$(-\infty, \infty)$	$[5, \infty)$
	SET BUILDER	$\{f(x) \mid f(x) \in \mathbb{R}\}$	$\{g(x) \mid g(x) \in \mathbb{R}\}$	$\{h(x) \mid h(x) \ge 5\}$

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	NOTATION	$f(x) = -2(x+7)^2 - 3$	$g(x) = 2(x+3) - 7$	$h(x) = -3(x+2)^3 - 7$
DOMAIN	INEQUALITY	$-\infty < x < \infty$		
	INTERVAL	$(-\infty, \infty)$		
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$		
RANGE	INEQUALITY	$-\infty < f(x) \leq -3$	$-\infty < g(x) < \infty$	$-\infty < h(x) < \infty$
	INTERVAL	$(-\infty, -3]$	$(-\infty, \infty)$	$(-\infty, \infty)$
	SET BUILDER	$\{f(x) \mid f(x) \leq -3\}$	$\{g(x) \mid g(x) \in \mathbb{R}\}$	$\{h(x) \mid h(x) \in \mathbb{R}\}$

The domains of all three of these functions contain all real numbers because the functions are polynomials, g(x) having degree one, f(x) having degree two, and h(x) having degree three. The ranges of g(x) and h(x) contain all real numbers since g(x) is a linear function and h(x) is a cubic function. The range of the quadratic function f(x) does not contain all real numbers but is restricted to only those real numbers less than or equal to the value of its parameter d, negative three, since the fact that its parameter a is negative indicates that the parabola opens downward.

YOU TRY IT! #4

Identify and compare the domains and ranges of $f(x) = \frac{1}{2}(x-4)^3 + 5$, $g(x) = \frac{1}{2}(x-4) + 5$, and $h(x) = \frac{1}{2}(x-4)^2 + 5$. Write the domain and range of each function as inequalities, as intervals, and in set builder notation.

See margin.

PRACTICE/HOMEWORK

For questions 1 - 8, describe the transformation of the cubic parent function, $f(x) = x^3$ that will result in the graph of the cubic function given.

1. $h(x) = (2x-1)^3$
See margin.

2. $g(x) = -3(x)^3 + 4$
See margin.

3. $h(x) = 2(x+2)^3$
See margin.

4. $g(x) = (-\frac{1}{4}x + 2)^3 + 5$
See margin.

5. $h(x) = -\frac{3}{4}(x-6)^3 + 3$
See margin.

6. $g(x) = \frac{1}{2}(4x+3)^3 - 2$
See margin.

7. $h(x) = -3(\frac{1}{4}x - 1)^3 + 5$
See margin.

8. $g(x) = -(-6x+5)^3 - 3$
See margin.

YOU TRY IT! #4 ANSWER:

See margin on bottom of page 180.

1. *The graph of h(x) is produced by transforming the cubic parent function f(x) by horizontally compressing its graph by a factor of one half and translating its graph one half unit to the right.*
2. *The graph of g(x) is produced by transforming the cubic parent function f(x) by vertically stretching its graph by a factor of three, reflecting its graph over the x-axis, and translating its graph four units up.*
3. *The graph of h(x) is produced by transforming the cubic parent function f(x) by vertically stretching its graph by a factor of two and translating its graph two units to the left.*
4. *The graph of g(x) is produced by transforming the cubic parent function f(x) by horizontally stretching its graph by a factor of four, reflecting its graph over the y-axis, and translating its graph eight units to the right and five units up*
5. *The graph of h(x) is produced by transforming the cubic parent function f(x) by vertically compressing its graph by a factor of three fourths, reflecting its graph over the x-axis, and translating its graph six units to the right and three units up*
6. *The graph of g(x) is produced by transforming the cubic parent function f(x) by vertically compressing its graph by a factor of one half, horizontally compressing its graph by a factor of one fourth, and translating its graph three-fourths unit to the left and two units down.*
7. *The graph of h(x) is produced by transforming the cubic parent function f(x) by vertically stretching its graph by a factor of three, horizontally stretching its graph by a factor of four, reflecting its graph over the x-axis, and translating its graph four units to the right and five units up.*
8. *The graph of g(x) is produced by transforming the cubic parent function f(x) by horizontally compressing its graph by a factor of one sixth, reflecting its graph over the x-axis, reflecting its graph over the y-axis, and translating its graph five-sixths unit to the right and three units down.*

9 *The domain of f(x) contains all real values of x. As an inequality, this is written as $-\infty < x < \infty$. In set builder notation, the domain is $\{x \mid x \in \mathbb{R}\}$.*

The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$. In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is $(-\frac{2}{3}, 0)$.

The y-intercept of f(x) is (0, 8).

10. *The domain of f(x) contains all real values of x. As an inequality, this is written as $-\infty < x < \infty$. In set builder notation, the domain is $\{x \mid x \in \mathbb{R}\}$.*

The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$. In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is (1.85, 0).

The y-intercept of f(x) is (0, 27).

11. *The domain of f(x) contains all real values of x. As an inequality, this is written as $-\infty < x < \infty$. In set builder notation, the domain is $\{x \mid x \in \mathbb{R}\}$.*

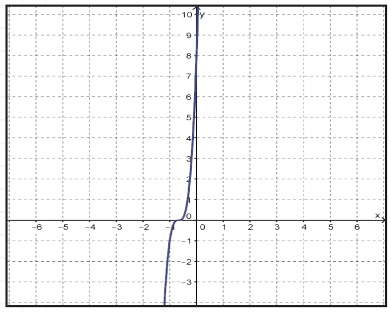
The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$. In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is (3.42, 0).

The y-intercept of f(x) is (0, 13).

For questions 9 - 11, identify the key attributes, including its domain, range, x-intercept(s), and y-intercepts of the linear function described by the equation and the graph. Write the domain and range as inequalities and in set builder notation.

9. $f(x) = (3x + 2)^3$



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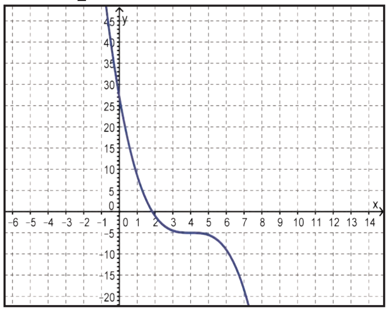
10. $f(x) = -\frac{1}{2}(x - 4)^3 - 5$



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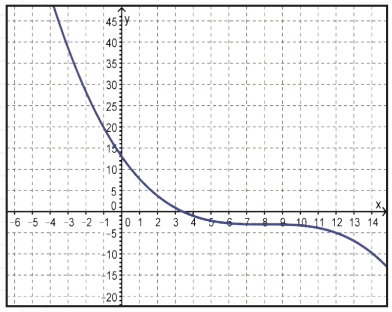
11. $f(x) = 2(-\frac{1}{4}x + 2)^3 - 3$



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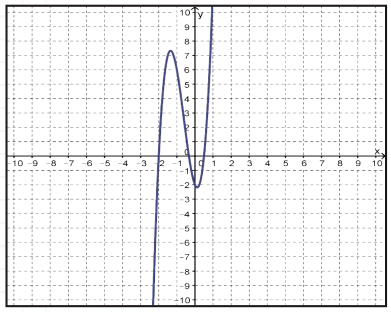
For questions 12 - 14, identify the key attributes, including its domain, range, relative minimum, relative maximum, x-intercept(s), and y-intercepts of the linear function described by the equation and the graph. Write the domain and range as intervals. Use graphing technology to determine the relative minimum, relative maximum, and x-intercepts.

12. $f(x) = (x + 2)(2x - 1)(3x + 1)$



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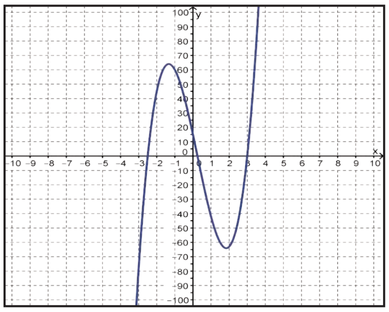
13. $f(x) = (2x + 5)(x - 3)(4x - 1)$



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12. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both $(-\infty, \infty)$.*

The cubic function has a relative minimum at (0.12, -2.19) and a relative maximum at (-1.34, 7.34)

The x-intercepts of f(x) are (-2, 0), (0.5, 0), $(-\frac{1}{3}, 0)$.

The y-intercept of f(x) is (0, -2)

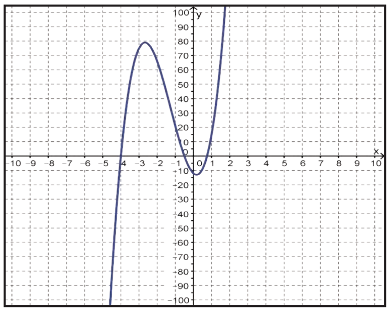
13. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both $(-\infty, \infty)$.*

The cubic function has a relative minimum at (1.84, -64.04) and a relative maximum at (-1.34, 64.04).

The x-intercepts of f(x) are (−2.5, 0), (3, 0), and (0.25, 0).

The y-intercept of f(x) is (0, 15).

14. $f(x) = (4x - 3)(2x + 1)(x + 4)$



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For questions 15 - 17, identify and compare the x-intercepts and the y-intercepts of the functions.

15. $f(x) = \frac{1}{2}(x - 4)$ $g(x) = \frac{1}{2}(x - 4)^2$ $h(x) = \frac{1}{2}(x - 4)^3$

See margin.

16. $f(x) = -2(x + 3) + 5$ $g(x) = -2(x + 3)^2 + 5$ $h(x) = -2(x + 3)^3 + 5$

See margin.

17. $f(x) = -3(2x + 5) + 9$ $g(x) = -3(2x + 5)^2 + 9$ $h(x) = -3(2x + 5)^3 + 9$

See margin.

For questions 18 - 20, complete the table to write the domain and range of each function as inequalities, as intervals, and in set builder notation.

18. $f(x) = (2x + 3) - 6$ $g(x) = (2x + 3)^2 - 6$ $h(x) = (2x + 3)^3 - 6$

	NOTATION	$f(x) = (2x + 3) - 6$	$g(x) = (2x + 3)^2 - 6$	$h(x) = (2x + 3)^3 - 6$
DOMAIN	INEQUALITY	$-\infty < x < \infty$		
	INTERVAL	$(-\infty, \infty)$		
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$		
RANGE	INEQUALITY	$-\infty < f(x) < \infty$	$-6 \leq g(x) < \infty$	$-\infty < h(x) < \infty$
	INTERVAL	$(-\infty, \infty)$	$[-6, \infty)$	$(-\infty, \infty)$
	SET BUILDER	$\{f(x) \mid f(x) \in \mathbb{R}\}$	$\{g(x) \mid g(x) \geq -6\}$	$\{h(x) \mid h(x) \in \mathbb{R}\}$

14. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both $(-\infty, \infty)$.*

The cubic function has a relative minimum at (0.17, -12.96) and a relative maximum at (-2.67, 78.96)

The x-intercepts of f(x) are (-4, 0), (0.75, 0), $(-\frac{1}{2}, 0)$.

The y-intercept of f(x) is (0, -12).

15. *The x-intercept of f(x) is (4, 0). The x-intercept of g(x) is (4, 0). The x-intercept of h(x) is (4, 0). The cubic function h(x) has only one x-intercept because the inflection point is on the x-axis. Since f(x) is a linear function, it has just one x-intercept. All three functions appear to have similar equations and their x-intercepts are the same. This happens for these functions because only the parameters a and c were changed.*

The y-intercept of f(x) is (0, -2). The y-intercept of g(x) is (0, 8).
The y-intercept of h(x) is (0, -32). Any function can have only one y-intercept. Although the equations that describe the functions are very similar, their y-intercepts are different because the parameters a, b, c, and d also affect the y-intercepts of linear, quadratic, and cubic functions differently.

16. *The x-intercept of f(x) is (-0.5, 0). The x-intercepts of g(x) are (-4.58, 0) and (-1.42, 0). The x-intercept of h(x) is (-1.64, 0). The cubic function h(x) has only one x-intercept instead because its inflection point is above the x-axis. Since f(x) is a linear function, it has just one x-intercept. Although all three functions appear to have similar equations, their x-intercepts are different because the parameters a, b, c, and d affect the x-intercepts of linear, quadratic, and cubic functions differently.*

The y-intercept of f(x) is (0, -1). The y-intercept of g(x) is (0, -13). The y-intercept of h(x) is (0, -49). Any function can have only one y-intercept. Although the equations that describe the functions are very similar, their y-intercepts are different because the parameters a, b, c, and d also affect the y-intercepts of linear, quadratic, and cubic functions differently.

17. *The x-intercept of f(x) is (-1, 0). The x-intercepts of g(x) are (-3.37, 0) and (-1.63, 0). The x-intercept of h(x) is (-1.78, 0). The cubic function h(x) has only one x-intercept instead because its inflection point is above the x-axis. Since f(x) is a linear function, it has just one x-intercept. Although all three functions appear to have similar equations, their x-intercepts are different because the parameters a, b, c, and d affect the x-intercepts of linear, quadratic, and cubic functions differently.*

The y-intercept of f(x) is (0, -6). The y-intercept of g(x) is (0, -66). The y-intercept of h(x) is (0, -366). Any function can have only one y-intercept. Although the equations that describe the functions are very similar, their y-intercepts are different because the parameters a, b, c, and d also affect the y-intercepts of linear, quadratic, and cubic functions differently.

19. $f(x) = -\frac{1}{2}(x+5)+3$ $g(x) = -\frac{1}{2}(x+5)^2+3$ $h(x) = -\frac{1}{2}(x+5)^3+3$

	NOTATION	$f(x) = -\frac{1}{2}(x+5)+3$	$g(x) = -\frac{1}{2}(x+5)^2+3$	$h(x) = -\frac{1}{2}(x+5)^3+3$
DOMAIN	INEQUALITY	$-\infty < x < \infty$		
	INTERVAL	$(-\infty, \infty)$		
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$		
RANGE	INEQUALITY	$-\infty < f(x) < \infty$	$-\infty < f(x) \leq 3$	$-\infty < h(x) < \infty$
	INTERVAL	$(-\infty, \infty)$	$(-\infty, 3]$	$(-\infty, \infty)$
	SET BUILDER	$\{f(x) \mid f(x) \in \mathbb{R}\}$	$\{g(x) \mid g(x) \leq 3\}$	$\{h(x) \mid h(x) \in \mathbb{R}\}$

20. $f(x) = 4(-\frac{1}{2}x-1)-8$ $g(x) = 4(-\frac{1}{2}x-1)^2-8$ $h(x) = 4(-\frac{1}{2}x-1)^3-8$

	NOTATION	$f(x) = 4(-\frac{1}{2}x-1)-8$	$g(x) = 4(-\frac{1}{2}x-1)^2-8$	$h(x) = 4(-\frac{1}{2}x-1)^3-8$
DOMAIN	INEQUALITY	$-\infty < x < \infty$		
	INTERVAL	$(-\infty, \infty)$		
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$		
RANGE	INEQUALITY	$-\infty < f(x) < \infty$	$g(x) \geq -8$	$-\infty < h(x) < \infty$
	INTERVAL	$(-\infty, \infty)$	$[-8, \infty)$	$(-\infty, \infty)$
	SET BUILDER	$\{f(x) \mid f(x) \in \mathbb{R}\}$	$\{g(x) \mid g(x) \geq -8\}$	$\{h(x) \mid h(x) \in \mathbb{R}\}$

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