2.2 Transforming and Analyzing Quadratic Functions

TEKS

AR.3A Compare and contrast the key attributes, including domain, range, maxima, minima, and intercepts, of a set of functions such as a set comprised of a linear, a quadratic, and an exponential function or a set comprised of an absolute value, a quadratic, and a square root function tabularly, graphically, and symbolically.

AR.7A Represent domain and range of a function using interval notation, inequalities, and set (builder) notation.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

5G Narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.

VOCABULARY

quadratic function, translation, reflection, compression, stretch, slope, *y*-intercept, *x*-intercept

MATERIALS

- graphing calculator

FOCUSING QUESTION How do transformations affect the domain, range, intercepts, and vertex of a quadratic function?

LEARNING OUTCOMES

- I can compare and contrast the key attributes of a quadratic function with other functions when I have the function as a table, graph, or written symbolically.
- I can represent the domain and range of a quadratic function in a variety of ways, including interval notation, inequalities, and set builder notation.
- I can use multiple representations, including symbols, graphs, tables, and language, to communicate mathematical ideas.

ENGAGE

Ethan designs rectangular fences. He knows that the area of a rectangle is found using the formula $A = lw$ and that the perimeter of a rectangle is found using the formula $P = 2l + 2w$. He rearranged these formulas to identify three quadratic functions that can be used to determine the area of a rectangle with a given perimeter.

- Rectangle *R* has a perimeter of 36 yards, so $A = w(18 - w)$.
- Rectangle *S* has a perimeter of 48 yards, so $A = w(24 - w)$.
- Rectangle *T* has a perimeter of 50 yards, so $A = w(25 - w)$.

For any given perimeter, there is a rectangle width, *w*, that will generate a maximum area. Which of these rectangles has the longest width for its maximum area? Justify your answer.

See margin.

EXPLORE

The general form of a quadratic function is $y = a(bx - c)^2 + d$, where *a*, *b*, *c*, and *d* represent real numbers. Use your graphing calculator to graph the four functions shown in each box on the same screen. Graph the first function, Y1, in bold or a different color. Use the graphs and tables of values on the graphing calculator to answer the questions next to the box.

ENGAGE ANSWER:

The rectangle with the greatest area is a square. For rectangle R the greatest area will be a 9 by 9 square. For rectangle S the greatest area will be a 12 by 12 square. For rectangle T the greatest area will be a 12.5 by 12.5 square. Therefore, Rectangle T has the longest width for its maximum area

INTEGRATE TECHNOLOGY

Encourage students to use a graphing calculator's table and graph features to compare the three functions. Look for critical points such as the vertex, *x*-intercepts, and *y*-intercept of each function. Students can use the graph to visually interpret the critical points on the quadratic function and compare the results. Doing so helps them focus on ways to work with quadratic functions that they will encounter in this lesson.

150 CHAPTER 2: ANALYZING FUNCTIONS

INVESTIGATING *a*

- $Y1 = (2x)^2$
- $Y2 = 2(2x)^2$
- $Y3 = 4(2x)^2$
- $Y4 = 6(2x)^2$

1. What happens to the graph of $y = a(2x)^2$ when the value of *a* increases?
See margin.

- $Y1 = (2x)^2$
- $Y2 = 0.5(2x)^2$
- $Y3 = 0.25(2x)^2$
- $Y4 = 0.1(2x)^2$

2. What happens to the graph of $y = a(2x)^2$ when the value of *a* is between 0 and 1?
See margin.

- $Y1 = (2x)^2$
- $Y2 = -(2x)^2$
- $Y3 = 3(2x)^2$
- $Y4 = -3(2x)^2$

3. What happens to the graph of $y = a(2x)^2$ when the value of *a* changes signs from positive to negative?
See margin.

INVESTIGATING *b*

- $Y1 = x^2$
- $Y2 = (2x)^2$
- $Y3 = (4x)^2$
- $Y4 = (10x)^2$

4. What happens to the graph of $y = (bx)^2$ when the value of *b* increases?
See margin.

- $Y1 = x^2$
- $Y2 = (0.5x)^2$
- $Y3 = (0.25x)^2$
- $Y4 = (0.1x)^2$

5. What happens to the graph of $y = (bx)^2$ when the value of *b* is between 0 and 1?
See margin.

- $Y1 = x^2$
- $Y2 = (-x)^2$
- $Y3 = (4x)^2$
- $Y4 = (-4x)^2$

6. What happens to the graph of $y = (bx)^2$ when the value of *b* changes signs from positive to negative?
See margin.

INVESTIGATING *c*

- $Y1 = (2x)^2$
- $Y2 = (2x - 3)^2$
- $Y3 = (2x - 5)^2$
- $Y4 = (2x - 8)^2$

7. What happens to the graph of $y = (bx - c)^2$ when the value of *c* increases?
See margin.

- $Y1 = (2x)^2$
- $Y2 = (2x + 2)^2$
- $Y3 = (2x + 3)^2$
- $Y4 = (2x + 6)^2$

8. What happens to the graph of $y = (bx - c)^2$ when the value of *c* is negative and decreases?
See margin.

STRATEGIES FOR SUCCESS

For quadratic functions, when *b* changes sign, the *y*-values do not experience a change in value because of the squaring function. $(b)^2 = (-b)^2$ because when you multiply a negative number by a negative number, the product is positive. Also, the parent quadratic function, $y = x^2$, has an axis of symmetry of $x = 0$, which is the *y*-axis.

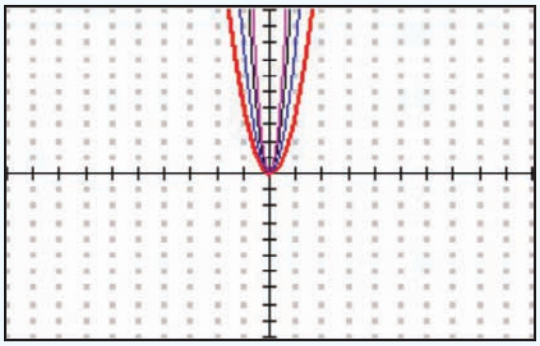
As a consequence, it may appear to students as though the graphs for *Y*1 and *Y*2 and for *Y*3 and *Y*4 are identical. Remind students of the rules of integer multiplication and discuss the axis of symmetry for a parabola.

The parameter *b* is a multiplier to the independent variable. When *b* is negative, the sign of the independent variable changes but the magnitude, or distance from the vertex, does not. This is how the parabola is reflected across the *y*-axis. Use a table of values to illustrate this phenomenon.

x	$3x$	$-3x$	$(-3x)^2$
-2	-6	6	36
-1	-3	3	9
0	0	0	0
1	3	-3	9
2	6	-6	36

Show students how the values for $3x$ are the opposite of the values for $-3x$ and how this generates a reflection across the *y*-axis prior to squaring the values for $(-3x)$.

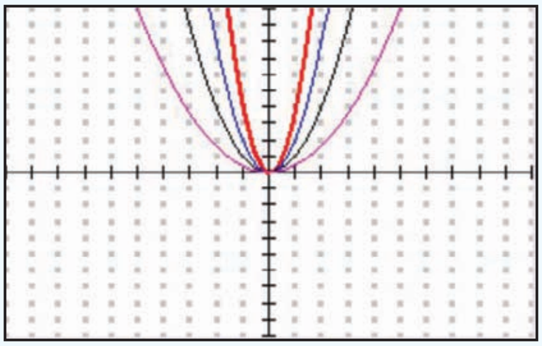
1. *As a increases, the parabola becomes vertically stretched because the y-values are moved farther from the x-axis.*



X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	4	8	16	24
2	16	32	64	96
3	36	72	144	216
4	64	128	256	384
5	100	200	400	600
6	144	288	576	864
7	196	392	784	1176
8	256	512	1024	1536
9	324	648	1296	1944
10	400	800	1600	2400

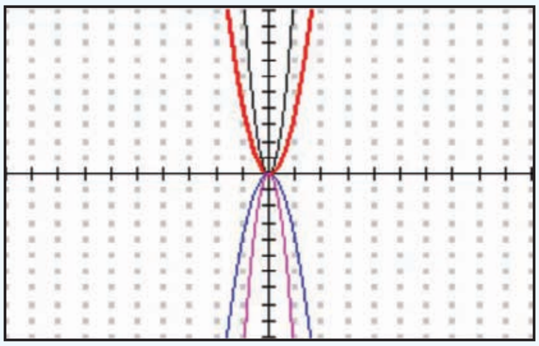
2-8. *See page 150-A and 151-A.*

2. *As the value of a gets closer to 0, the parabola becomes more vertically compressed because the y-values are moved closer to the x-axis.*



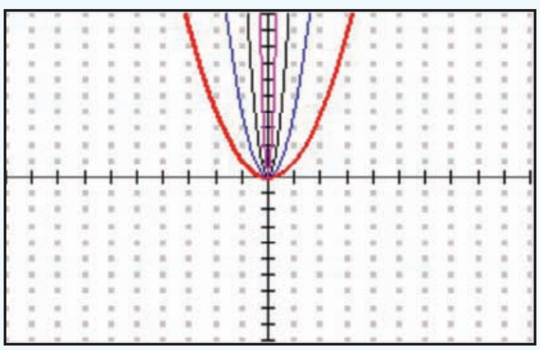
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	4	2	1	.4
2	16	8	4	1.6
3	36	18	9	3.6
4	64	32	16	6.4
5	100	50	25	10
6	144	72	36	14.4
7	196	98	49	19.6
8	256	128	64	25.6
9	324	162	81	32.4
10	400	200	100	40

3. *When a changes signs, the graph is reflected across the x-axis.*



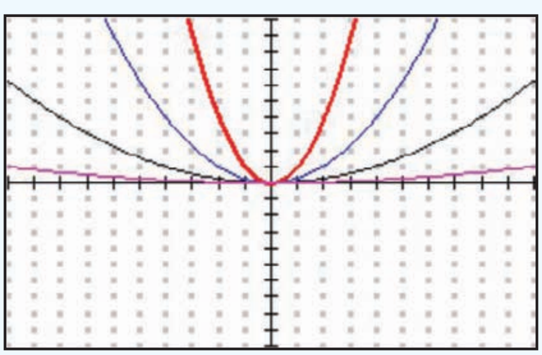
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	4	-4	12	-12
2	16	-16	48	-48
3	36	-36	108	-108
4	64	-64	192	-192
5	100	-100	300	-300
6	144	-144	432	-432
7	196	-196	588	-588
8	256	-256	768	-768
9	324	-324	972	-972
10	400	-400	1200	-1200

4. *As b increases, the parabola becomes more horizontally compressed because the x-values are moved closer to the y-axis.*



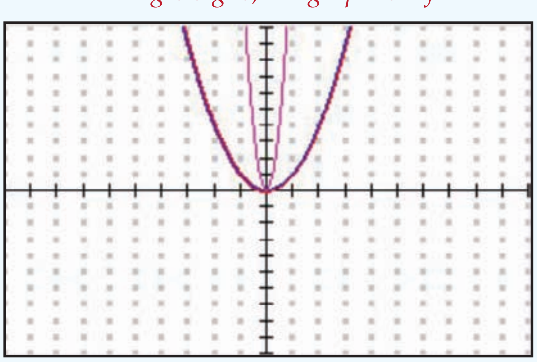
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	1	4	16	100
2	4	16	64	400
3	9	36	144	900
4	16	64	256	1600
5	25	100	400	2500
6	36	144	576	3600
7	49	196	784	4900
8	64	256	1024	6400
9	81	324	1296	8100
10	100	400	1600	10000

5. *As the value of b gets closer to 0, the parabola becomes more horizontally stretched because the x-values are moved farther from the y-axis.*



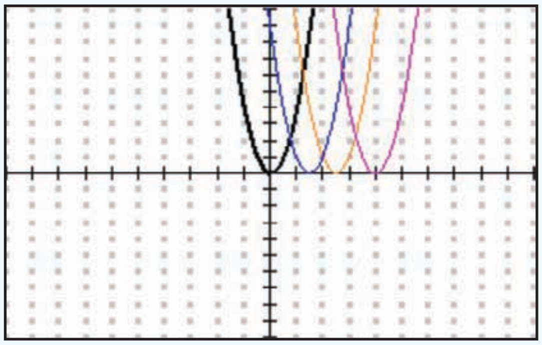
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	1	.25	.0625	.01
2	4	1	.25	.04
3	9	2.25	.5625	.09
4	16	4	1	.16
5	25	6.25	1.5625	.25
6	36	9	2.25	.36
7	49	12.25	3.0625	.49
8	64	16	4	.64
9	81	20.25	5.0625	.81
10	100	25	6.25	1

6. *When b changes signs, the graph is reflected across the y-axis.*



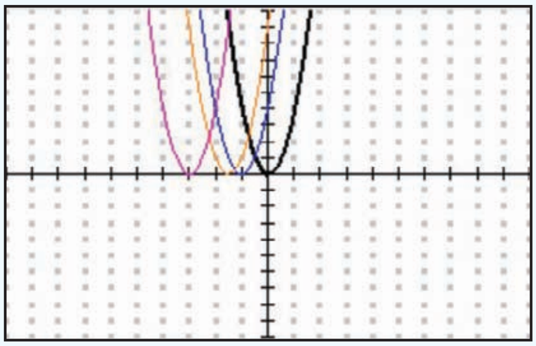
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	1	1	16	16
2	4	4	64	64
3	9	9	144	144
4	16	16	256	256
5	25	25	400	400
6	36	36	576	576
7	49	49	784	784
8	64	64	1024	1024
9	81	81	1296	1296
10	100	100	1600	1600

7. *As c increases, the parabola shifts or translates $\frac{c}{b}$ units to the right from the parent function.*



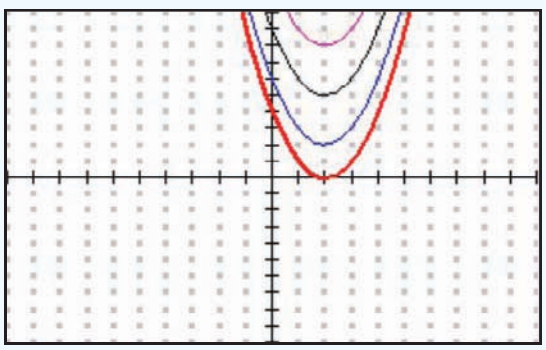
X	Y1	Y2	Y3	Y4
0	0	9	25	64
1	4	1	9	36
2	16	1	1	16
3	36	9	1	4
4	64	25	9	0
5	100	49	25	4
6	144	81	49	16
7	196	121	81	36
8	256	169	121	64
9	324	225	169	100
10	400	289	225	144

8. *As c decreases, the parabola shifts or translates $\frac{c}{b}$ units to the left of the parent function.*



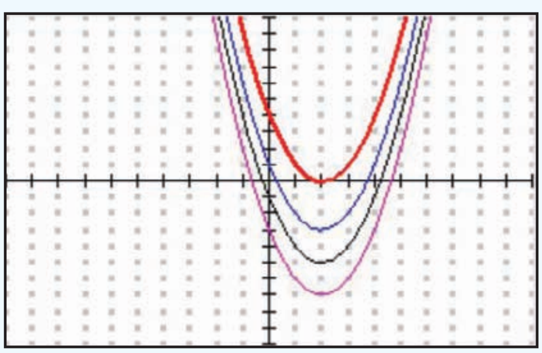
X	Y1	Y2	Y3	Y4
0	0	4	9	36
1	4	16	25	64
2	16	36	49	100
3	36	64	81	144
4	64	100	121	196
5	100	144	169	256
6	144	196	225	324
7	196	256	289	400
8	256	324	361	484
9	324	400	441	576
10	400	484	529	676

9. *As d increases, the line shifts or translates d units upward from the original function.*



X	Y1	Y2	Y3	Y4
0	4	6	9	12
1	1	3	6	9
2	0	2	5	8
3	1	3	6	9
4	4	6	9	12
5	9	11	14	17
6	16	18	21	24
7	25	27	30	33
8	36	38	41	44
9	49	51	54	57
10	64	66	69	72

10. *As d decreases, the line shifts or translates d units downward from the original function.*



X	Y1	Y2	Y3	Y4
0	4	1	-1	-3
1	1	-2	-4	-6
2	0	-3	-5	-7
3	1	-2	-4	-6
4	4	1	-1	-3
5	9	6	4	2
6	16	13	11	9
7	25	22	20	18
8	36	33	31	29
9	49	46	44	42
10	64	61	59	57

11. *The domain of the quadratic parent function, $y = x^2$, is all real numbers. The parameters a, b, c, and d do not affect the domain of the quadratic function.*

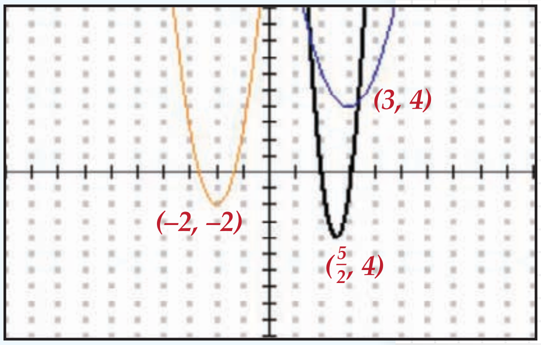
12. *The range of the quadratic parent function, $y = x^2$, is all real numbers greater than or equal to 0: $\{y \mid y \geq 0\}$. The parameters affecting horizontal transformations, b and c, do not affect the range. However, the parameters affecting vertical transformations, a and d, do affect the range.*

The parameter d changes the range of the function $y = a(bx - c)^2 + d$ from $\{y \mid y \geq 0\}$ to $\{y \mid y \geq d\}$.

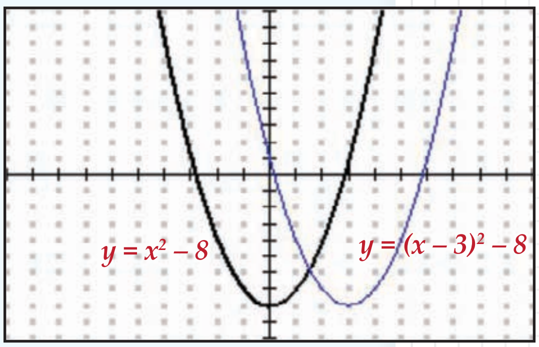
The parameter a, if it is negative, changes the range of the function $y = a(bx - c)^2 + d$ to $\{y \mid y \leq d\}$.

9–12. *See page 151-A.*

13.



15.



16. *The original y-intercept is (0, −8). The parameter c translates the parabola 3 units to the right, which translates the y-intercept $3^2 = 9$ units upward to (0, 1).*

The parameter c does not affect the y-intercept since it dilates the parabola with respect to the y-axis.

The parameter a dilates the parabola by a factor of 2, so a doubles the distance between the original y-intercept, (0, −8), and the transformed y-intercept, (0, 10), from 9 units before a is applied to 2 × 9 = 18 units after a is applied.

INVESTIGATING d

- $Y1 = (x - 2)^2$
- $Y2 = (x - 2)^2 + 2$
- $Y3 = (x - 2)^2 + 5$
- $Y4 = (x - 2)^2 + 8$

- $Y1 = (x - 2)^2$
- $Y2 = (x - 2)^2 - 3$
- $Y3 = (x - 2)^2 - 5$
- $Y4 = (x - 2)^2 - 7$

9. What happens to the graph of $y = (x - 2)^2 + d$ when the value of d increases?
See margin.

10. What happens to the graph of $y = (x - 2)^2 + d$ when the value of d is negative and decreases?
See margin.

Use your investigations to answer the following questions.

11. How do changes in the parameters a, b, c, and d affect the domain of a quadratic function?
See margin.

12. How do changes in the parameters a, b, c, and d affect the range of a quadratic function?
See margin.

13. Use a graphing calculator to graph the functions below. What do you notice about the relationship between the values of a, b, c, and d and the location of the vertex?

- $y = 3(2x - 5)^2 - 4$ **vertex ($\frac{5}{2}$, −4)**
- $y = 2(x - 3)^2 + 4$ **vertex (3, 4)**
- $y = 0.5(3x + 6)^2 - 2$ **vertex (−2, −2)**

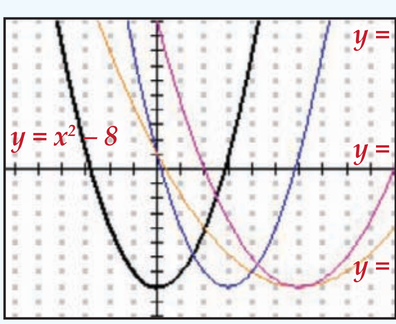
The vertex moves from (0, 0) for the quadratic parent function $\frac{c}{b}$ units horizontally and d units vertically. See margin for graph.

14. How could you write the coordinates of the vertex using the parameters a, b, c, and d?
The vertex is the point ($\frac{c}{b}$, d).

15. Graph the functions $y = x^2 - 8$ and $y = (x - 3)^2 - 8$. How does the y-intercept of each function compare to the values of c and d?
The value of d moves the y-intercept vertically d units and the value of c moves the y-intercept upward c^2 units. See margin for graph.

16. Add the functions $y = (0.5x - 3)^2 - 8$ and $y = 2(0.5x - 3)^2 - 8$ to your graph. How do the values of a and b affect the y-intercept?
See margin.

17. How could you write the coordinates of the y-intercept using the parameters a, b, c, and d?
The y-intercept is the point (0, $ac^2 + d$)



DIFFERENTIATING INSTRUCTION

Marian Small and Amy Lin (*More Good Questions: Great Ways to Differentiate Mathematics*) recommend using parallel tasks and open questions as differentiation structures to make mathematical tasks accessible to all students. Parallel tasks are two tasks that accomplish the same learning outcome with different levels of complexity. Students select one of the two tasks based on their comfort level with the topic. In this lesson, parallel tasks that could be posed for the Reflect questions include:

The equation of a quadratic function is $y = a(bx - c)^2 + d$.

OPTION 1: If the graph of the function passes through the origin, what do you know about a, b, c, and d?

OPTION 2: If the graph of the function has no x-intercepts, what do you know about a, b, c, and d?

REFLECT

- **The parameters *a* and *b* are multiplicative parameters. How do changes in *a* and *b* affect the graph of $y = x^2$?**
  See margin.
- **The parameters *c* and *d* are additive parameters. How do changes in *c* and *d* affect the graph of $y = x^2$?**
  See margin.
- **Which parameters affect the domain and range of a quadratic function? Why do you think that is the case?**
  See margin.
- **Which parameters affect the vertex of a quadratic function? Why do you think that is the case?**
  See margin.
- **Which parameters affect the *y*-intercept of a quadratic function? Why do you think that is the case?**
  See margin.

EXPLAIN

A **quadratic function** is a relationship between an independent variable, usually x, and a dependent variable, usually y or $f(x)$, that generates a graph in the shape of a parabola.

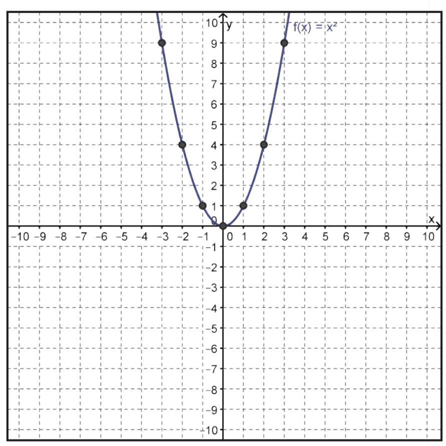
Watch Explain and You Try It Videos

or click here

You can represent a quadratic function using a table of values, graph, or symbols. The quadratic parent function, $f(x) = x^2$, is shown in the table, graph, and symbolic representation. The table contains a set of points from the domain and range of the quadratic parent function. These points are plotted on the graph.

$f(x) = x^2$

x	$f(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



The full family of quadratic functions is generated by applying transformations to the quadratic parent function. Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship.

REFLECT ANSWERS:

The parameter a generates a vertical dilation or a reflection across the x-axis. If $a > 1$, then the graph is vertically stretched by a factor of a. If $0 < a < 1$, then the graph is vertically compressed by a factor of a. If $a < 0$, then the graph is reflected across the x-axis.

The parameter b generates a horizontal dilation or a reflection across the y-axis. If $b > 1$, then the graph is horizontally compressed by a factor of $\frac{1}{b}$. If $0 < b < 1$, then the graph is horizontally stretched by a factor of $\frac{1}{b}$. If $b < 0$, then the graph is reflected across the y-axis.

The parameter c generates a horizontal translation. If $c > 0$, then the graph is horizontally translated c units to the right. If $c < 0$, then the graph is horizontally translated c units to the left.

The parameter c works with the parameter b to generate a horizontal translation. If $c > 0$, then the graph is horizontally translated $|\frac{c}{b}|$ units to the right. If $c < 0$, then the graph is horizontally translated $|\frac{c}{b}|$ units to the left.

The domain for all quadratic functions is all real numbers, so a parameter change does not affect the domain. The range is affected by d, which vertically shifts the vertex of the parabola, which is the maximum or minimum value of the quadratic function, and by a if there is a reflection across the x-axis.

REFLECT ANSWERS CONTINUED:

The x-coordinate of the vertex is affected by the parameters b and c, which influence horizontal dilations and horizontal translations. The y-coordinate of the vertex is affected by the parameter d, which influences vertical translations.

The y-intercept is affected by the parameters a, c, and d. The parameters c and d influence horizontal and vertical translations which move the vertex of the graph. The parameter a influences vertical dilations and reflections which stretch, compress, or reflect the y-intercept. The parameter b does not affect the y-intercept because horizontal dilations are performed with respect to the y-axis, which has a value of $x = 0$. Multiplying b by 0 does not change the y-coordinate of the y-intercept.

INSTRUCTIONAL HINT

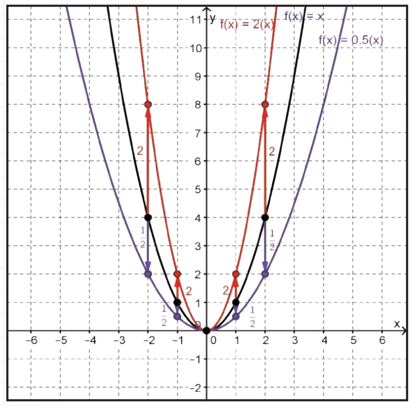
Have students continue the graphic organizer of transformations they began in section 2.1. Compare and contrast changes in a, b, c, and d for linear and quadratic functions.

For a quadratic function, the general form is $f(x) = a(bx - c)^2 + d$, where a, b, c, and d are real numbers.

CHANGES IN a

The parameter a influences the vertical dilation of the parabola, or graph of the quadratic function.

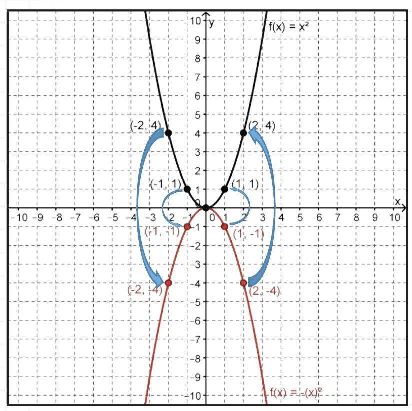
- If $a > 1$, then the range values (y-coordinates) of the original function are multiplied by a factor of a in order to vertically stretch the parabola.
- If $0 < a < 1$, then the range values (y-coordinates) of the original function are multiplied by a factor of a in order to vertically compress the parabola.



x	$f(x) = x^2$	$f(x) = 2(x)^2$	$f(x) = 0.5(x)^2$
-2	4	8	2
-1	1	2	0.5
0	0	0	0
1	1	2	0.5
2	4	8	2

× 2 × 0.5

The parameter a also affects the orientation of the parabola. If $a < 0$, then the parabola will be reflected across the x-axis.



x	$f(x) = (x)^2$	$f(x) = -(x)^2$
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4

× -1

CHANGES IN b

The parameter b influences the horizontal stretch or compression of the parabola.

- If $b > 1$, then the domain values (x-coordinates) of the original function are multiplied by a factor of $\frac{1}{b}$, which will be a multiplier that is less than 1, in order to horizontally compress the parabola.

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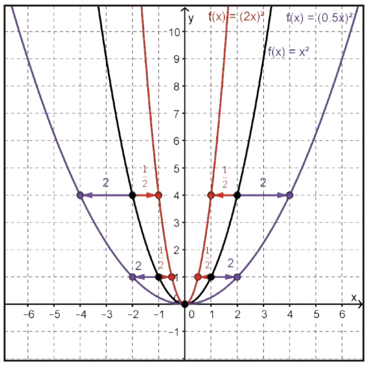
- If $0 < b < 1$, then the domain values (x-coordinates) of the original function are multiplied by a factor of $\frac{1}{b}$, which will be a multiplier that is greater than 1, in order to horizontally stretch the parabola.



x	$f(x) = x^2$	x	$f(x) = (2x)^2$
-2	4	-1	4
-1	1	$-\frac{1}{2}$	1
0	0	0	0
1	1	$\frac{1}{2}$	1
2	4	1	4

Domain (x) values are multiplied by $\frac{1}{2}$ in order to generate the same range (y) value. This multiplication results in a horizontal compression of the graph.

x	$f(x) = x^2$	x	$f(x) = (0.5x)^2$
-2	4	-4	4
-1	1	-2	1
0	0	0	0
1	1	2	1
2	4	4	4

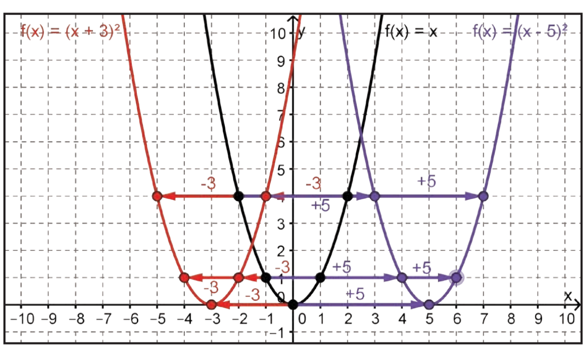
Domain (x) values are multiplied by 2 in order to generate the same range (y) value. This multiplication results in a horizontal stretch of the graph.

The parameter b also affects the orientation of the parabola. If $b < 0$, then all of the x-values will change signs and the parabola will be reflected across the y-axis. However, because a parabola has a vertical axis of symmetry, the horizontal reflection is not noticeable in the graph.

CHANGES IN *c*

The parameters b and c influence the horizontal translation of the parabola. Notice that in the general form, $f(x) = a(bx - c)^2 + d$, the sign in front of c is negative. That means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation. For example, $f(x) = 1.5(3x - 7)^2 + 1$ has $c = 7$ and $f(x) = 1.5(3x + 7)^2 + 1$ has $c = -7$.

- If $c > 0$, then the graph of the line will translate $|\frac{c}{b}|$ units to the right.
- If $c < 0$, then the graph of the line will translate $|\frac{c}{b}|$ units to the left.



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x	$f(x) = x^2$
-2	4
-1	1
0	0
1	1
2	4

x	$f(x) = (x - 5)^2$
3	4
4	1
5	0
6	1
7	4

Domain (x) values are increased by 5 in order to generate the same range (y) value. This addition results in a horizontal translation of the graph to the right.

x	$f(x) = x^2$
-2	4
-1	1
0	0
1	1
2	4

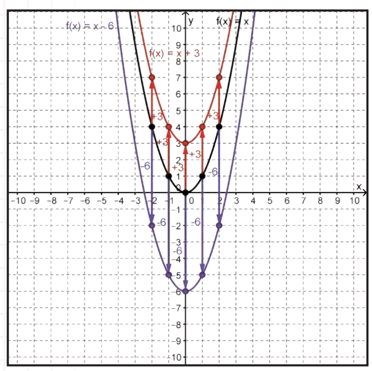
x	$f(x) = (x + 3)^2$
-5	4
-4	1
-3	0
-2	1
-1	4

Domain (x) values are decreased by 3 in order to generate the same range (y) value. This subtraction (addition with a negative number) results in a horizontal translation of the graph to the left.

CHANGES IN d

The parameter d influences the vertical translation of the graph of the parabola.

If $d > 0$, then the parabola will translate $|d|$ units upward.
If $d < 0$, then the parabola will translate $|d|$ units downward.



x	$f(x) = x^2$	$f(x) = x^2 - 6$	$f(x) = x^2 + 3$
-2	4	-2	7
-1	1	-5	4
0	0	-6	3
1	1	-5	4
2	4	-2	7

-6 +3

Each of these parameters also has an effect on key attributes of a quadratic function.

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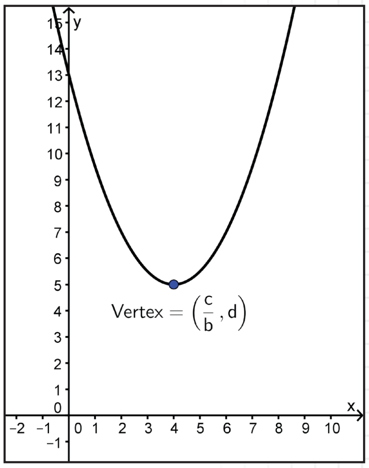
VERTEX

The **vertex of a parabola** is a maximum or minimum value. If the parabola opens upward, then the vertex is the minimum value for the function. If the parabola opens downward, then the vertex is the maximum value.



Three parameters influence the location of the vertex: *b*, *c*, and *d*. If you start with the general equation for a quadratic function, $f(x) = a(bx - c)^2 + d$, the equation $m = 2a(bx - c)$ allows you to calculate the slope of a line that is tangent to the parabola through any point with a known *x*-coordinate.

For a vertex, which is a maximum or minimum point, the tangent line is horizontal, so its slope should be 0. Set the slope equation equal to 0 in order to calculate the *x*-coordinate of the vertex.

A line that is **tangent** to a graph or shape touches the graph or shape at only one point.

$$0 = 2a(bx - c)$$
$$\frac{0}{2a} = \frac{2a(bx - c)}{2a}$$
$$0 = bx - c$$
$$0 + c = bx - c + c$$
$$c = bx$$
$$\frac{c}{b} = \frac{bx}{b}$$
$$\frac{c}{b} = x$$

To calculate the *y*-coordinate of the vertex, substitute $x = \frac{c}{b}$ into the general equation, $y = a(bx - c) + d$.

$$y = a\left(b\left(\frac{c}{b}\right) - c\right) + d$$
$$y = a(c - c) + d$$
$$y = a(0) + d$$
$$y = d$$

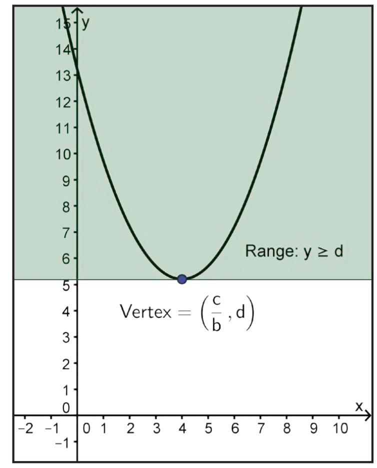
The *x*-coordinate of the vertex is $\frac{c}{b}$ and the *y*-coordinate of the vertex is *d*, so the vertex is the ordered pair $\left(\frac{c}{b}, d\right)$.

DOMAIN AND RANGE

A quadratic function involves squaring a number. Every real number can be squared, so there are no domain restrictions on quadratic functions. The domain is always all real numbers, or $\{x \mid x \in \mathbb{R}\}$.

The range of a quadratic function, however, does have restrictions. The sign of a squared number is always positive because according to the rules of integer operations, when you multiply two numbers together with the same sign, the product is always positive.

The range is affected by the parameters d and a. If the parabola opens upward, as is the case when $a > 0$, then the value of d sets the y-coordinate of the vertex at a minimum value and the range is $y \geq$ d.



If the parabola opens downward, as is the case when $a < 0$, then the value of d sets the y-coordinate of the vertex at a maximum value, and the range is $y \leq d$.

y-INTERCEPT

The y-intercept of any function is the point $(0, y)$, or the point where the graph intersects the y-axis. For a quadratic function, you can calculate the y-intercept by substituting $x = 0$ into the general form, $y = a(bx - c)^2 + d$.

$$y = a(b(0) - c)^2 + d$$
$$y = a(0 - c)^2 + d$$
$$y = a(-c)^2 + d$$
$$y = ac^2 + d$$

The parameters a, c, and d each influence the y-intercept of a quadratic function. The parameter d vertically translates the entire parabola, including the y-intercept, and the parameter c horizontally translates the entire parabola, including the y-intercept. The parameter a vertically dilates or reflects the entire parabola, including the y-intercept. Each of these transformations affects where the parabola intersects the y-axis.

KEY ATTRIBUTES OF QUADRATIC FUNCTIONS

A quadratic function has several important key attributes:

- The domain of a quadratic function is all real numbers.
- The range of a quadratic function is $\{y | y \geq d\}$ for $a > 0$ and $\{y | y \leq d\}$ for $a < 0$.
- The vertex of a quadratic function is $\left(\frac{c}{b}, d\right)$. If $a > 0$ the vertex is a minimum value. If $a < 0$, the vertex is a maximum value.
- A quadratic function has as many as two x-intercepts that are the same as the zeros of the function. The x-intercepts are located at $\left(\frac{c \pm \sqrt{\frac{-d}{a}}}{b}, 0\right)$.
- A quadratic function has one y-intercept at $(0, ac^2 + d)$.

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EXAMPLE 1

What transformations of the quadratic parent function, $f(x) = x^2$, will result in the graph of the quadratic function $g(x) = \frac{1}{3}(x - 1)^2 - 4$?

STEP 1 **Rewrite the equation of *g(x)* in general form to determine the values of the parameters *a*, *b*, *c* and *d*.**

$$g(x) = a(bx - c)^2 + d$$
$$g(x) = \frac{1}{3}(x - 1)^2 - 4$$
$$g(x) = \frac{1}{3}(x - 1)^2 + (-4)$$

Therefore, $a = \frac{1}{3}$, $b = 1$, $c = 1$, and $d = -4$.

STEP 2 **Use the values of the parameters to describe the transformations of the quadratic parent function *f(x)* that are necessary to produce *g(x)*.**

$a = \frac{1}{3}$, so $0 < a < 1$. The range values (y-coordinates) of the quadratic parent function are multiplied by a factor of $\frac{1}{3}$ in order to vertically compress the graph of the line.

$b = 1$, which has no transformational effect on the graph.

$c = 1$, so $c > 0$. The graph of the quadratic parent function will translate $|1|$ unit to the right.

$d = -4$, so $d < 0$. The graph of the quadratic parent function will translate $|-4| = 4$ units downward.

The graph of g(x) is produced by transforming the quadratic parent function f(x) by vertically compressing its graph by a factor of one third and translating its graph one unit to the right and four units downward.

YOU TRY IT! #1

What transformations of the quadratic parent function, $y = x^2$, will result in the graph of the quadratic function $y = 2(x + 3)^2$?

See margin.

ADDITIONAL EXAMPLES

What transformations of the quadratic parent function, $f(x) = x^2$, will result in the graph of the quadratic functions below?

1. $h(x) = -2(3x + 10)^2 + 3$

The graph of h(x) is produced by transforming the quadratic parent function f(x) by vertically stretching its graph by a factor of 2, horizontally compressing its graph by a factor of one third, reflecting its graph over the x-axis, and translating its graph ten thirds units to the left and three units up.

2. $p(x) = \frac{5}{3}(-\frac{1}{5}x - 4)^2 - 6$

The graph of p(x) is produced by transforming the quadratic parent function f(x) by vertically stretching its graph by a factor of five thirds, horizontally stretching its graph by a factor of five, reflecting its graph over the y-axis, and translating its graph left twenty units and down six units.

YOU TRY IT! #1 ANSWER:

The graph of $y = 2(x + 3)^2$ is produced by transforming the quadratic parent function by vertically stretching its graph by a factor of two and translating its graph three units to the left.

ADDITIONAL EXAMPLES

Identify the key attributes of the functions below including domain, range, vertex, x-intercept(s), and y-intercept. Write the domain and range as intervals and in set builder notation. Determine whether the vertex is a maximum or a minimum value of the function.

1. $y = -3\frac{1}{6}(x - 9)^2 - 7$

The domain is $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$. The range is $(-\infty, -7]$ which may also be written as $\{y \mid y \leq -7\}$. The vertex of the parabola is (9, -7) and it is the maximum value. This quadratic function has no x-intercept, and the y-intercept is (0, -250).

2. $y = \frac{1}{2}(-10x + 4)^2 + 6$

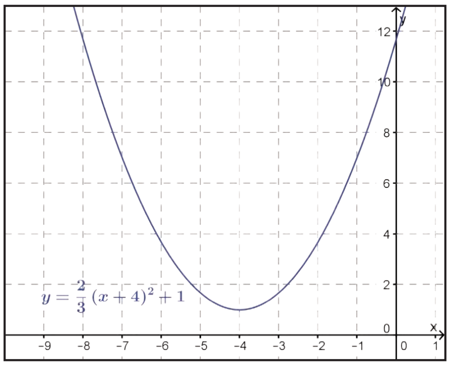
The domain is $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$. The range is $[6, \infty)$ which may also be written as $\{y \mid y \geq 6\}$. The vertex of the parabola is $(\frac{2}{5}, 6)$ and it is the minimum value. This quadratic function has no x-intercept, and the y-intercept is (0, 14).

3. $y = 6(2x - 10)^2 - 3$

The domain is $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$. The range is $[-3, \infty)$ which may also be written as $\{y \mid y \geq -3\}$. The vertex of the parabola is (5, -3) and it is the minimum value. The x-intercepts are (5.35, 0) and (4.65, 0), and the y-intercept is (0, 597).

EXAMPLE 2

Identify the key attributes of $y = \frac{2}{3}(x + 4)^2 + 1$, including domain, range, vertex, x-intercept(s), and y-intercept. Write the domain and range as intervals and in set builder notation. Determine whether the vertex is a maximum or a minimum value of the function.



STEP 1 **Determine the domain and range of $y = \frac{2}{3}(x + 4)^2 + 1$.**

From the graph, the domain contains all real values of x. As an interval, this is written as $(-\infty, \infty)$. The same information about the domain written in set builder notation is $\{x \mid x \in \mathbb{R}\}$.

Using the equation to determine the range, the value of a indicates that the parabola opens upward. This is confirmed by the graph. The only other parameter that affects the range of the quadratic function is d. Since $d = 1$, the range of this quadratic function contains all real values of y that are greater than or equal to one. As an interval, this is written as $[1, \infty)$. The same information about the range written in set builder notation is $\{y \mid y \geq 1\}$.

STEP 2 **Determine the vertex of the parabola.**

The vertex of a quadratic function written in the general form $y = a(bx - c)^2 + d$ is $\left(\frac{c}{b}, d\right)$.

$\frac{c}{b} = \frac{-4}{1} = -4$ and $d = 1$.

The vertex of the parabola is (– 4, 1). Since the value of a is positive, the vertex is a minimum. This is confirmed by the graph.

STEP 3 **Determine the x-intercept(s) of the quadratic function $y = \frac{2}{3}(x + 4)^2 + 1$.**

If they exist, the x-intercepts are located at $\left(\frac{c \pm \sqrt{\frac{-d}{a}}}{b}, 0\right)$.

$$\frac{c \pm \sqrt{\frac{-d}{a}}}{b} = \frac{-4 \pm \sqrt{\frac{-1}{\frac{2}{3}}}}{1} = \frac{-4 \pm \sqrt{\frac{-3}{2}}}{1}$$

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Because the square root of a negative number is not a real number, the quadratic function $y = \frac{2}{3}(x + 4)^2 + 1$ does not have an x-intercept. The graph confirms this, since the graph lies completely above the x-axis and does not touch or cross the x-axis. The fact that the range does not include zero also confirms that this quadratic function does not have an x-intercept.

STEP 4 **Determine the y-intercept of $f(x)$.**

y-intercepts of functions occur where the domain or input value $x = 0$.

$y = \frac{2}{3}(x + 4)^2 + 1$

$y = \frac{2}{3}(0 + 4)^2 + 1 = \frac{2}{3}(4)^2 + 1 = \frac{2}{3}(16) + 1 = \frac{32}{3} + 1 = \frac{35}{3} = 11\frac{2}{3}$.

The y-intercept of the quadratic function is $(0, 11\frac{2}{3})$. The graph confirms this since the line intersects the y-axis at a point between (0, 11) and (0, 12).

The domain of $y = \frac{2}{3}(x + 4)^2 + 1$ is $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$. The range is $[1, \infty)$ which may also be written as $\{y \mid y \geq 1\}$. The vertex of the parabola is $(-4, 1)$ and it is the minimum function value. This quadratic function has no x-intercept, and the y-intercept is $(0, 11\frac{2}{3})$.

YOU TRY IT! #2

Identify the key attributes of $f(x) = (4x - 1)^2 - 2$, including its domain, range, vertex, x-intercept(s), and y-intercepts. Write the domain and range of $f(x)$ as inequalities and in set builder notation. Determine whether the vertex is a maximum or minimum value.

x	$f(x)$
– 3	167
– 2	79
– 1	23
0	– 1
1	7
2	47
3	119

See margin.

QUESTIONING STRATEGY

In section 2.1, students used interval notation for domain and range of linear functions. The domain and range was all real numbers. The range for quadratics includes a numerical value.

- Why are the numerical values in the range of quadratic functions written with square brackets in interval notation?

YOU TRY IT! #2 ANSWER:

The domain of $f(x)$ is $-\infty < x < \infty$, which can also be written as $\{x \mid x \in \mathbb{R}\}$. The range of $f(x)$ is $[-2, \infty)$ or $\{f(x) \mid f(x) \geq -2\}$. The vertex of the parabola is $(\frac{1}{4}, -2)$, and it is the minimum function value. The x-intercepts are $(\frac{1 + \sqrt{2}}{4}, 0)$ and $(\frac{1 - \sqrt{2}}{4}, 0)$. The y-intercept is $(0, -1)$.

ADDITIONAL EXAMPLES

Identify and compare the x-intercept(s) of the linear and quadratic functions below.

1. $f(x) = 3(x + 4) - 2.5$

$g(x) = 3(x + 4)^2 - 2.5$

The x-intercept of $f(x)$ is (-3.17, 0). The x-intercepts of $g(x)$ are (-3.09, 0) and (-4.91, 0)

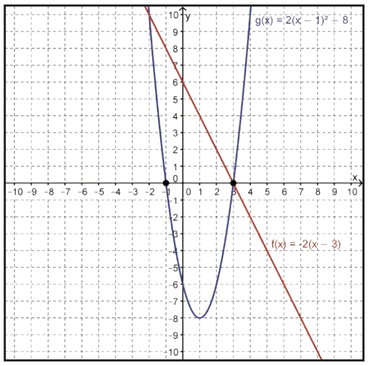
2. $f(x) = -\frac{1}{2}(-4x - 6) + 2$

$g(x) = 2(x - \frac{1}{2})^2 - 8$

The x-intercept of $f(x)$ is (-2.5, 0). The x-intercepts of $g(x)$ are (2.5, 0) and (-1.5, 0)

EXAMPLE 3

Identify and compare the x-intercept(s) of $f(x) = -2(x - 3)$ and the x-intercept(s) of $g(x) = 2(x - 1)^2 - 8$.



STEP 1 **Graph both $f(x)$ and $g(x)$ to visually estimate the x-intercepts.**

STEP 2 **Determine the x-intercept(s) of $f(x)$.**

Since $f(x)$ is a linear function, $f(x)$ has one x-intercept at $(\frac{ac - d}{ab}, 0)$.

$$\frac{ac - d}{ab} = \frac{(-2)(3) - 0}{(-2)(1)} = \frac{-6}{-2} = 3$$

The x-intercept of $f(x)$ is (3, 0).

STEP 3 **Determine the x-intercept(s) of $g(x)$.**

Since $f(x)$ is a quadratic function, the x-intercepts are located at $\left(\frac{c \pm \sqrt{\frac{-d}{a}}}{b}, 0\right)$.

$$\frac{c \pm \sqrt{\frac{-d}{a}}}{b} = \frac{1 \pm \sqrt{\frac{-(-8)}{2}}}{1} = \frac{1 \pm \sqrt{\frac{8}{2}}}{1} = \frac{1 \pm \sqrt{4}}{1} = \frac{1 \pm 2}{1}$$

$$\frac{1 + 2}{1} = \frac{3}{1} = 3$$

$$\frac{1 - 2}{1} = \frac{-1}{1} = -1$$

The x-intercepts of $g(x)$ are (–1, 0) and (3, 0).

STEP 4 **Compare the x-intercept of $f(x)$ and the x-intercepts of $g(x)$.**

Since $f(x)$ is a linear function, it has only one x-intercept, (3, 0). The quadratic function $g(x)$ has two x-intercepts, (-1, 0) and (3, 0). One of the intercepts of $g(x)$ is the same as the x-intercept of $f(x)$.

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YOU TRY IT! #3

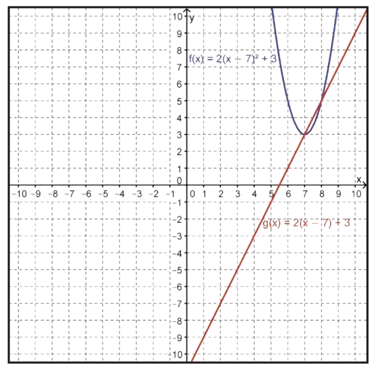
Identify and compare the x-intercept(s) of $f(x) = -\frac{1}{2}(x-5)^2$ and the x-intercept(s) of $g(x) = \frac{3}{5}(x-15)$.

See margin.

EXAMPLE 4

Identify and compare the domain and range of $f(x) = 2(x-7)^2 + 3$ and the domain and range of $g(x) = 2(x-7) + 3$. Write the domain and range of each function as inequalities, as intervals, and in set builder notation.

STEP 1 Graph both $f(x)$ and $g(x)$ to visually estimate the x-intercepts.



STEP 2 Determine the domain and range of $f(x)$.

DOMAIN

Since $f(x)$ is a quadratic function, the domain contains all real values of x. The domain can be expressed using one of three types of notation.

- Interval notation: $(-\infty, \infty)$
- Set builder notation: $\{x \mid x \in \mathbb{R}\}$
- Inequality: $-\infty < x < \infty$

RANGE

Using the equation, the value of a indicates that the parabola opens upward. The only other parameter that affects the range of a quadratic function is d. Since $d = 3$, the range of this quadratic function contains all real values of $f(x)$ that are greater than or equal to three. The range can be expressed using one of three types of notation.

- Interval notation: $[3, \infty)$
- Set builder notation: $\{f(x) \mid f(x) \geq 3\}$
- Inequality: $3 \leq f(x) < \infty$

YOU TRY IT! #3 ANSWER:

The x-intercept of $f(x)$ is (5, 0). The x-intercept of $g(x)$ is (15, 0). The quadratic function $f(x)$ has only one x-intercept instead of two because its vertex is on the x-axis rather than above or below the x-axis. Since $g(x)$ is a linear function, it has just one x-intercept.

ADDITIONAL EXAMPLE

Identify and compare the domain and range of the functions below. Write the domain and range of each function as inequalities, as intervals, and in set builder notation.

$f(x) = -\frac{2}{3}(3x + 6) - 7$

$g(x) = -\frac{2}{3}(3x + 6)^2 - 7$

The domains of both these functions contain all real numbers.

- *Inequality $-\infty < x < \infty$*
- *Interval: $(-\infty, \infty)$*
- *Set Builder: $\{x \mid x \in \mathbb{R}\}$*

The range of $f(x)$ does not contain all real numbers but is restricted to only those real numbers less than or equal to -7.

- *Inequality: $-\infty < f(x) \leq -7$*
- *Interval: $(-\infty, -7]$*
- *Set Builder: $\{f(x) \mid f(x) \leq -7\}$*

The range of $g(x)$ contains all real numbers.

- *Inequality: $-\infty < g(x) < \infty$*
- *Interval: $(-\infty, \infty)$*
- *Set Builder: $\{g(x) \mid g(x) \in \mathbb{R}\}$*

STEP 3 **Determine the domain and range of $g(x)$.**

DOMAIN

Since $g(x)$ is a linear function, its domain contains all real values of x. The domain can be expressed using one of three types of notation.

- Interval notation: $(-\infty, \infty)$
- Set builder notation: $\{x \mid x \in \mathbb{R}\}$
- Inequality: $-\infty < x < \infty$

RANGE

Since $g(x)$ is a linear function, its range contains all real values of $g(x)$. The range can be expressed using one of three types of notation.

- Interval notation: $(-\infty, \infty)$
- Set builder notation: $\{g(x) \mid g(x) \in \mathbb{R}\}$
- Inequality: $-\infty < g(x) < \infty$

STEP 4 **Compare the domains of $f(x)$ and $g(x)$ and the ranges of $f(x)$ and $g(x)$.**

The domains of both these functions are the same because both functions are polynomials, one of degree one and the other of degree two. The range of $g(x)$ contains all real numbers since $g(x)$ is a linear function. The range of the quadratic function $f(x)$ does not contain all real numbers but is restricted to only those real numbers greater than or equal to three.

	NOTATION	$f(x) = 2(x-7)^2 + 3$	$g(x) = 2(x-7) + 3$
DOMAIN	INEQUALITY	$-\infty < x < \infty$	
	INTERVAL	$(-\infty, \infty)$	
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$	
RANGE	INEQUALITY	$3 \le f(x) < \infty$	$-\infty < g(x) < \infty$
	INTERVAL	$[3, \infty)$	$(-\infty, \infty)$
	SET BUILDER	$\{f(x) \mid f(x) \ge 3\}$	$\{g(x) \mid g(x) \in \mathbb{R}\}$

The range of the quadratic function $f(x)$ does not contain all real numbers but is restricted to only those real numbers greater than or equal to the value of its parameter d. Even though the equations of the functions look very similar, they do not have the same range.

YOU TRY IT! #4 ANSWER:

The domains of both these functions contain all real numbers because both functions are polynomials, $f(x)$ having degree one and $g(x)$ having degree two. The range of $f(x)$ contains all real numbers since $f(x)$ is a linear function. The range of the quadratic function $g(x)$ does not contain all real numbers but is restricted to only those real numbers less than or equal to the value of its parameter d. The reason the range of $g(x)$ contains values less than or equal to negative two rather than greater or equal to is that the value of its parameter a is negative.

	NOTATION	$f(x) = (0.5x + 1) - 2$	$g(x) = -0.5(x + 1)^2 - 2$
DOMAIN	INEQUALITY	$-\infty < x < \infty$	
	INTERVAL	$(-\infty, \infty)$	
	SET BUILDER	$\{x \mid x \in \mathbb{R}\}$	
RANGE	INEQUALITY	$-\infty < f(x) < \infty$	$-\infty < g(x) \le -2$
	INTERVAL	$(-\infty, \infty)$	$(-\infty, -2]$
	SET BUILDER	$\{f(x) \mid f(x) \in \mathbb{R}\}$	$\{g(x) \mid g(x) \le -2\}$

YOU TRY IT! #4

Identify and compare the domain and range of $f(x) = (0.5x + 1) - 2$ and the domain and range of $g(x) = -0.5(x + 1)^2 - 2$. Write the domain and range of each function as inequalities, as intervals, and in set builder notation.

See margin.

PRACTICE/HOMEWORK

1. For the quadratic function $y = a(bx - c)^2 + d$, which of the parameter values (a, b, c, or d) will produce the transformation described?

- horizontal stretch or compression **b**
- a translation upward or downward **d**
- vertical stretch or compression **a**
- a translation left or right **c**

For questions 2 - 7, describe what transformations of the quadratic parent function, $f(x) = x^2$ will result in the graph of the given function.

2. $g(x) = 2(x - 3)^2$
See margin.

3. $h(x) = -\frac{1}{4}(x)^2 + 5$
See margin.

4. $g(x) = (4x - 7)^2$
See margin.

5. $h(x) = \left(\frac{1}{2}x\right)^2 - 1$
See margin.

6. $g(x) = -3(x + 2)^2 + 6$
See margin.

7. $h(x) = \frac{1}{3}(2x - 5)^2 - 4$
See margin.

8. The graph of $g(x)$ is produced by transforming the quadratic parent function, $f(x) = x^2$, by vertically stretching its graph by a factor of 3 and translating it 7.5 units upward. Determine the equation that represents $g(x)$.
$g(x) = 3(x)^2 + 7.5$

9. The graph of $h(x)$ is produced by transforming the quadratic parent function, $f(x) = x^2$, by reflecting its graph over the x-axis, and translating it 3 units to the left and 11 units downward. Determine the equation that represents $h(x)$.
$h(x) = -(x + 3)^2 - 11$

For each quadratic function given in questions 10 – 12 identify the vertex, and determine whether it is a maximum or a minimum value.

10. $g(x) = -(x - 1.5)^2 - 4$
See margin.

11. $g(x) = -3(4x)^2 + 7$
See margin.

12. $g(x) = (2x - 5)^2 + 3$
See margin.

YOU TRY IT! #4 ANSWER:
See margin on bottom of page 164.

2. *Transform the quadratic parent function, f(x), by vertically stretching its graph by a factor of 2, and translating its graph 3 units to the right.*

3. *Transform the quadratic parent function, f(x), by reflecting its graph over the x-axis, vertically compressing its graph by a factor of $\frac{1}{4}$, and translating its graph 5 units upward.*

4. *Transform the quadratic parent function, f(x), by horizontally compressing its graph by a factor of $\frac{1}{4}$, and translating its graph 1.75 units to the right.*

5. *Transform the quadratic parent function, f(x), by horizontally stretching its graph by a factor of 2, and translating its graph 1 unit downward.*

6. *Transform the quadratic parent function, f(x), by reflecting its graph over the x-axis, vertically stretching its graph by a factor of 3, and translating its graph 2 units to the left and 6 units upward.*

7. *Transform the quadratic parent function, f(x), by vertically compressing its graph by a factor of $\frac{1}{3}$, horizontally compressing its graph by a factor of $\frac{1}{2}$, and translating its graph 2.5 units to the right and 4 units downward.*

10. *The vertex is a maximum at (1.5, -4).*

11. *The vertex is a maximum at (0, 7)*

12. *The vertex is a minimum at $(\frac{5}{2}, 3)$, or (2.5, 3)*

13. *Domain: $-\infty < x < \infty$ or $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$.*
Range: $-\infty < h(x) \le -1$ or $(-\infty, -1]$ or $\{y \mid y \le -1\}$.
x-intercepts: none
y-intercept: (0, -10)
vertex: (6, -1)

14. *Domain: $-\infty < x < \infty$ or $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$.*
Range: $-\infty < f(x) \le 9$ or $(-\infty, 9]$ or $\{y \mid y \le 9\}$.
x-intercepts: (-4, 0) and (2, 0)
y-intercept: (0, 8)
vertex: (-1, 9)

15. *Domain: $-\infty < x < \infty$ or $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$.*
Range: $-8 \le g(x) < \infty$ or $(-8, \infty]$ or $\{y \mid y \ge -8\}$.
x-intercepts: (1, 0) and (-3, 0)
y-intercept: (0, -6)
vertex: (-1, -8)

13-15. *See bottom of page 165.*

16. *Domain: $-\infty < x < \infty$ or $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$.*
Range: $-5 \le g(x) < \infty$ or $[-5, \infty)$ or $\{y \mid y \ge -5\}$.
x-intercepts: $(-3 + \sqrt{\frac{5}{2}}, 0)$ and $(-3 - \sqrt{\frac{5}{2}}, 0)$ or (-1.42, 0) and (-4.58, 0)
y-intercept: (0, 13)
vertex: (-3, -5)

17. *Domain: $-\infty < x < \infty$ or $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$.*
Range: $-9 \le y < \infty$ or $[-9, \infty)$ or $\{y \mid y \ge -9\}$.
x-intercepts: (-1, 0) and (5, 0)
y-intercept: (0, -5)
vertex: (2, -9)

18. *Since f(x) is a linear function, it has only one x-intercept, (3, 0). The quadratic function g(x) has two x-intercepts, (3, 0) and (-1, 0). One of the x-intercepts of g(x) is the same as the x-intercept of f(x).*

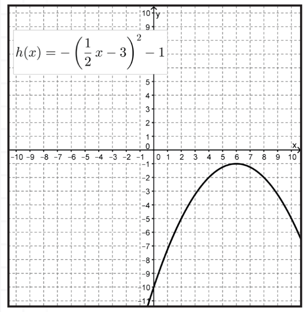
19. *Since h(x) is a linear function, it has only one x-intercept, (-8, 0). The quadratic function f(x) also has one x-intercept at (-6, 0), the vertex.*

20. *Both h(x) and g(x) have one y-intercept. The y-intercept of h(x) is (0, 17), while the y-intercept of g(x) is (0, 5).*

21. *Both f(x) and g(x) have one y-intercept. The y-intercept of both f(x) is (0, -5) a while the y-intercept of g(x) is (0, -1).*

For questions 13 – 17 identify the domain, range, x-intercept(s), y-intercept, and vertex of each quadratic function. Write the domain and range in three different ways: as an inequality, interval, and in set-builder notation.

13.



See margin.

14.

x	$f(x) = -(x+1)^2 + 9$
-5	-7
-4	0
-3	5
-2	8
-1	9
0	8
1	5
2	0

See margin.

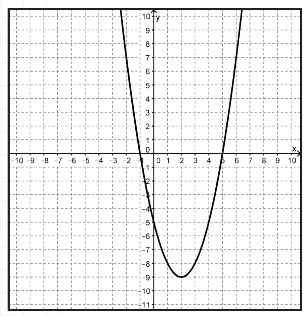
15. $g(x) = 2(x+1)^2 - 8$
See margin.

16.

x	$g(x) = 2(x+3)^2 - 5$
-5	3
-4	-3
-3	-5
-2	-3
-1	3
0	13
1	27

See margin.

17.



See margin.

For questions 18 – 19 identify and compare the x-intercepts of the following sets of functions:

18. $f(x) = 3(x - 3)$ and $g(x) = (x - 1)^2 - 4$
See margin.

19. $f(x) = -(x + 6)^2$ and $h(x) = -0.5(x + 6) - 1$
See margin.

For questions 20 – 21 identify and compare the y-intercepts of the following sets of functions:

20. $h(x) = 2(x + 3)^2 - 1$ and $g(x) = 2(x + 3) - 1$
See margin.

21. $f(x) = \frac{1}{3}(x - 6) - 3$ and $g(x) = -4(x - \frac{1}{2})^2$
See margin.

For questions 22 – 23 identify and compare the domain and range of the following sets of functions:

22. $g(x) = \frac{1}{2}(x + 4)^2 + 3$ and $f(x) = \frac{1}{2}(x + 4) + 3$
See margin.

23. $h(x) = -(2x - 1)^2 + 6$ and $f(x) -(2x - 1) + 6$
See margin.

22. *The domain of both g(x) and f(x) contain all real values of x:*

Inequality: $-\infty < x < \infty$ Interval: $(-\infty, \infty)$

Set Builder: $\{x \mid x \in \mathbb{R}\}$

Because f(x) is a linear function, its range contains all real values:

Inequality: $-\infty < f(x) < \infty$ Interval: $(-\infty, \infty)$

Set Builder: $\{f(x) \mid f(x) \in \mathbb{R}\}$

The value of a in the quadratic function g(x) tells us that the parabola opens upward. The d value of 3 in the equation indicates that the function contains all real values of g(x) that are greater than or equal to 3:

Inequality: $3 \le g(x) < \infty$ Interval: $[3, \infty)$

Set Builder: $\{g(x) \mid g(x) \ge 3\}$

23. *The domain of both h(x) and f(x) contain all real values of x:*

Inequality: $-\infty < x < \infty$ Interval: $(-\infty, \infty)$

Set Builder: $\{x \mid x \in \mathbb{R}\}$

Because f(x) is a linear function, its range contains all real values:

Inequality: $-\infty < f(x) < \infty$ Interval: $(-\infty, \infty)$

Set Builder: $\{f(x) \mid f(x) \in \mathbb{R}\}$

The value of a in the quadratic function h(x) tells us that the parabola opens downward. The d value of 6 in the equation indicates that the function contains all real values of h(x) that are less than or equal to 6:

Inequality: $-\infty < h(x) \le 6$ Interval: $(-\infty, 6]$

Set Builder: $\{h(x) \mid h(x) \le 6\}$

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