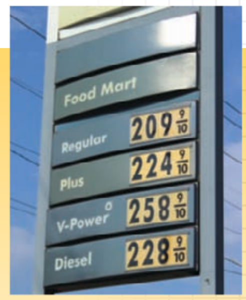
2.1 Transforming and Analyzing Linear Functions

FOCUSING QUESTION How does changing the slope or the y-intercept affect the graph of a linear function?

LEARNING OUTCOMES

- I can compare and contrast the key attributes of a linear function with other linear functions when I have the function as a table, graph, or written symbolically.
- I can represent the domain and range of a linear function in a variety of ways, including interval notation, inequalities, and set builder notation.
- I can use multiple representations, including symbols, graphs, tables, and language to communicate mathematical ideas.

ENGAGE



Minerva noticed that the total cost of gasoline, y, is a function of the number of gallons of gasoline, x, that she pumps into her fuel tank. Based on the prices of the three grades of gasoline at her neighborhood gas station, she wrote the following functions.

- $y = 2.19x$
- $y = 2.39x$
- $y = 2.65x$

If Minerva were to graph each function, which line would have the steepest slope? How can you tell?

See margin.

EXPLORE

The general form of a linear equation is $y = a(bx - c) + d$, where a, b, c, and d represent real numbers. Use your graphing calculator to graph the four functions shown in each box on the same screen. Graph the first function, $Y1$, in bold or a different color. Use the graphs and tables of values on the graphing calculator to answer the questions next to the box.

INVESTIGATING b

- $Y1 = x$
- $Y2 = 2x$
- $Y3 = 4x$
- $Y4 = 10x$

1. What happens to the graph of $y = bx$ when the value of b increases?

See margin.

TEKS

AR.3A Compare and contrast the key attributes, including domain, range, maxima, minima, and intercepts, of a set of functions such as a set comprised of a linear, a quadratic, and an exponential function or a set comprised of an absolute value, a quadratic, and a square root function tabularly, graphically, and symbolically.

AR.7A Represent domain and range of a function using interval notation, inequalities, and set (builder) notation.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

5B Write using newly acquired basic vocabulary and content-based grade-level vocabulary.

VOCABULARY

linear function, translation, reflection, compression, stretch, slope, y-intercept, x-intercept

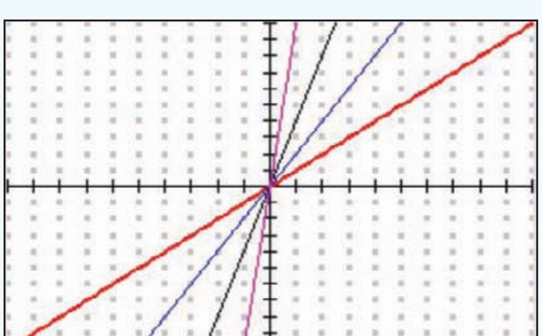
MATERIALS

- graphing calculator

ENGAGE ANSWER:

The graph of y = 2.65x would have the steepest slope because the rate of change, $2.65 per gallon, is greater than $2.39 or $2.19 per gallon.

1. *As b increases, the line becomes more vertical or steeper because the points with the same x-value are moved closer to the y-axis.*



X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	1	2	4	10
2	2	4	8	20
3	3	6	12	30
4	4	8	16	40
5	5	10	20	50
6	6	12	24	60
7	7	14	28	70
8	8	16	32	80
9	9	18	36	90
10	10	20	40	100

INTEGRATE TECHNOLOGY

Encourage students to use a graphing calculator's table and graph features to compare the three functions. Students may remember from previous courses that the greater the coefficient of x, the steeper the slope of the graph of the line. Use tables to see how the function values increase at a faster rate for functions with a greater coefficient of x or graphs to see how the line has a steeper slope from left to right. Multiple representations help students see different ways that the coefficient of x, which is the rate of change or slope in a linear function, affects the function itself. Technology makes these multiple representations accessible to all learners.

3-7. *See page 138-A.*

8-9. *See page 139-A.*

- $Y1 = x$
- $Y2 = 0.5x$
- $Y3 = 0.25x$
- $Y4 = 0.1x$

2. What happens to the graph of $y = bx$ when the value of b is between 0 and 1?
See margin.

- $Y1 = x - 2$
- $Y2 = -x - 2$
- $Y3 = 4x - 2$
- $Y4 = -4x - 2$

3. What happens to the graph of $y = bx$ when the value of b changes signs from positive to negative? *(Hint: For this set, graph Y1 and Y3 in bold, and if possible, Y1 and Y2 in one color and Y3 and Y4 in a second color).*
See margin.

INVESTIGATING a

- $Y1 = 2x$
- $Y2 = 2(2x)$
- $Y3 = 4(2x)$
- $Y4 = 10(2x)$

4. What happens to the graph of $y = a(2x)$ when the value of a increases?
See margin.

- $Y1 = 2x$
- $Y2 = 0.5(2x)$
- $Y3 = 0.25(2x)$
- $Y4 = 0.1(2x)$

5. What happens to the graph of $y = a(2x)$ when the value of a is between 0 and 1?
See margin.

- $Y1 = 2x - 6$
- $Y2 = -(2x - 6)$
- $Y3 = 3(2x - 6)$
- $Y4 = -3(2x - 6)$

6. What happens to the graph of $y = a(2x)$ when the value of a changes signs from positive to negative?
See margin.

INVESTIGATING c

- $Y1 = 2x$
- $Y2 = (2x - 4)$
- $Y3 = (2x - 6)$
- $Y4 = (2x - 7)$

7. What happens to the graph of $y = bx - c$ when the value of c increases?
See margin.

- $Y1 = 2x$
- $Y2 = (2x + 3)$
- $Y3 = (2x + 5)$
- $Y4 = (2x + 6)$

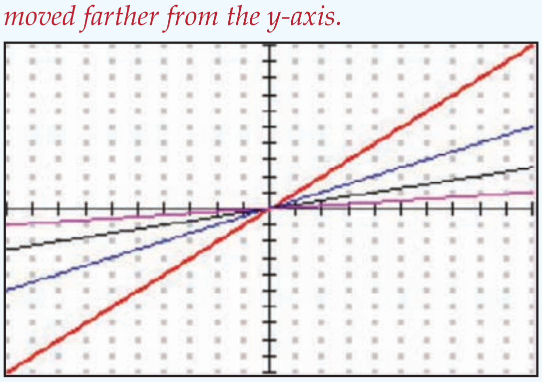
8. What happens to the graph of $y = bx - c$ when the value of c is negative and decreases?
See margin.

INVESTIGATING d

- $Y1 = (x - 2)$
- $Y2 = (x - 2) + 3$
- $Y3 = (x - 2) + 5$
- $Y4 = (x - 2) + 7$

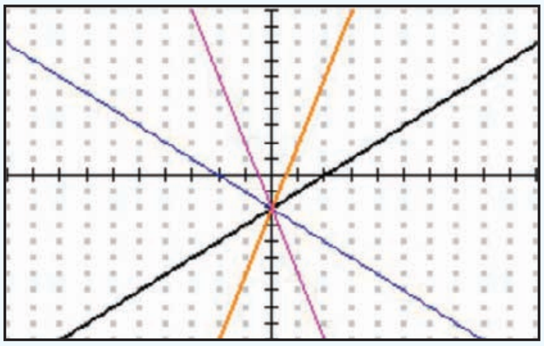
9. What happens to the graph of $y = (x - 2) + d$ when the value of d increases?
See margin.

2. *As b the value of b gets closer to 0, the line becomes more horizontal or less steep because the points with the same x-value are moved farther from the y-axis.*



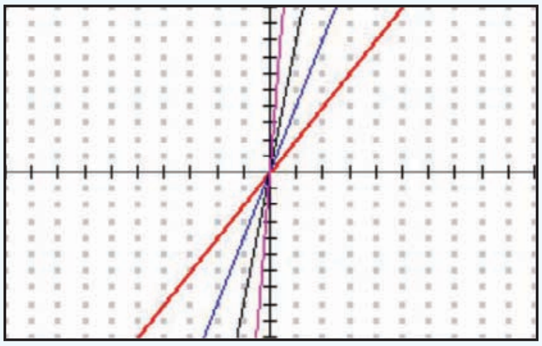
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	1	.5	.25	.1
2	2	1	.5	.2
3	3	1.5	.75	.3
4	4	2	1	.4
5	5	2.5	1.25	.5
6	6	3	1.5	.6
7	7	3.5	1.75	.7
8	8	4	2	.8
9	9	4.5	2.25	.9
10	10	5	2.5	1

3. *When b changes signs, the graph is reflected across the y-axis.*



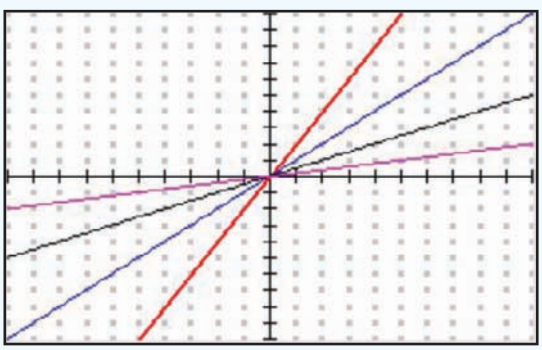
X	Y1	Y2	Y3	Y4
0	-2	-2	-2	-2
1	-1	-3	2	-6
2	0	-4	6	-10
3	1	-5	10	-14
4	2	-6	14	-18
5	3	-7	18	-22
6	4	-8	22	-26
7	5	-9	26	-30
8	6	-10	30	-34
9	7	-11	34	-38
10	8	-12	38	-42

4. *As a increases, the line becomes more vertical or steeper because the points with the same y-value are moved farther from the x-axis.*



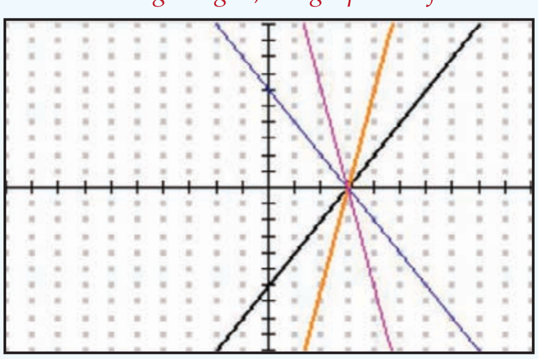
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	2	4	8	20
2	4	8	16	40
3	6	12	24	60
4	8	16	32	80
5	10	20	40	100
6	12	24	48	120
7	14	28	56	140
8	16	32	64	160
9	18	36	72	180
10	20	40	80	200

5. *As the value of a gets closer to 0, the line becomes more horizontal or less steep because the points with the same y-value are moved farther from the y-axis.*



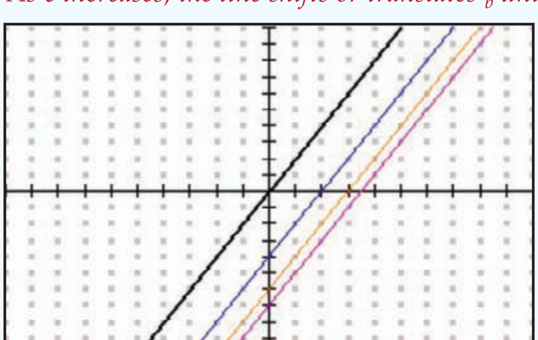
X	Y1	Y2	Y3	Y4
0	0	0	0	0
1	2	1	.5	.2
2	4	2	1	.4
3	6	3	1.5	.6
4	8	4	2	.8
5	10	5	2.5	1
6	12	6	3	1.2
7	14	7	3.5	1.4
8	16	8	4	1.6
9	18	9	4.5	1.8
10	20	10	5	2

6. *When a changes signs, the graph is reflected across the x-axis.*



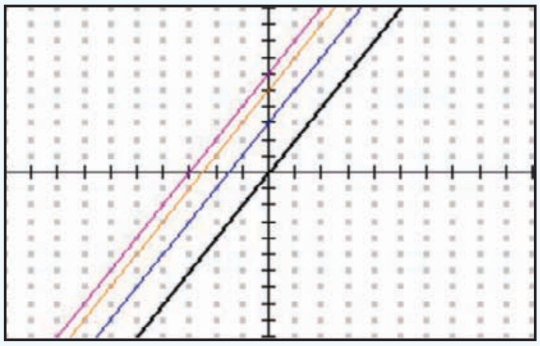
X	Y1	Y2	Y3	Y4
0	-6	6	-18	18
1	-4	4	-12	12
2	-2	2	-6	6
3	0	0	0	0
4	2	-2	6	-6
5	4	-4	12	-12
6	6	-6	18	-18
7	8	-8	24	-24
8	10	-10	30	-30
9	12	-12	36	-36
10	14	-14	42	-42

7. *As c increases, the line shifts or translates $\frac{c}{b}$ units to the right from the parent function.*



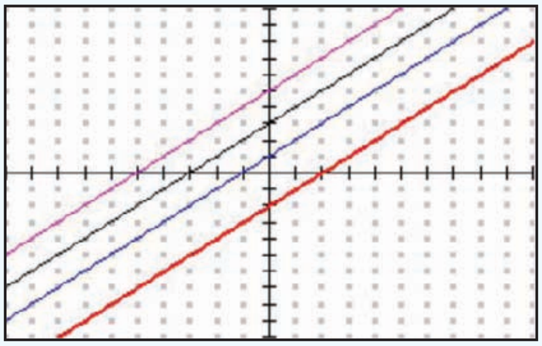
X	Y1	Y2	Y3	Y4
0	0	-4	-6	-7
1	2	-2	-4	-5
2	4	0	-2	-3
3	6	2	0	-1
4	8	4	2	1
5	10	6	4	3
6	12	8	6	5
7	14	10	8	7
8	16	12	10	9
9	18	14	12	11
10	20	16	14	13

8. *As c decreases, the line shifts or translates $\frac{c}{b}$ units to the left of the parent function.*



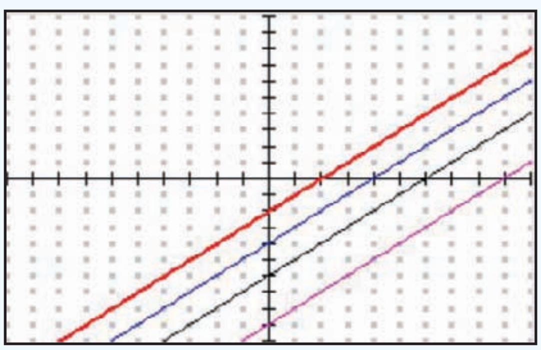
X	Y1	Y2	Y3	Y4
0	0	3	5	6
1	2	5	7	8
2	4	7	9	10
3	6	9	11	12
4	8	11	13	14
5	10	13	15	16
6	12	15	17	18
7	14	17	19	20
8	16	19	21	22
9	18	21	23	24
10	20	23	25	26

9. *As d increases, the line shifts or translates d units up from the original function.*



X	Y1	Y2	Y3	Y4
0	-2	1	3	5
1	-1	2	4	6
2	0	3	5	7
3	1	4	6	8
4	2	5	7	9
5	3	6	8	10
6	4	7	9	11
7	5	8	10	12
8	6	9	11	13
9	7	10	12	14
10	8	11	13	15

10. *As d decreases, the line shifts or translates d units down from the original function.*



X	Y1	Y2	Y3	Y4
0	-2	-4	-6	-9
1	-1	-3	-5	-8
2	0	-2	-4	-7
3	1	-1	-3	-6
4	2	0	-2	-5
5	3	1	-1	-4
6	4	2	0	-3
7	5	3	1	-2
8	6	4	2	-1
9	7	5	3	0
10	8	6	4	1

- $Y1 = x$
- $Y2 = (x - 2) - 2$
- $Y3 = (x - 2) - 4$
- $Y4 = (x - 2) - 7$

10. What happens to the graph of $y = (x - 2) + d$ when the value of d is negative and decreases?
See margin.

REFLECT

- **In general, how does the parameter *a* affect the graph of $y = a(bx - c) + d$?**
  See margin.
- **In general, how does the parameter *b* affect the graph of $y = a(bx - c) + d$?**
  See margin.
- **In general, how does the parameter *c* affect the graph of $y = a(bx - c) + d$?**
  See margin.
- **In general, how does the parameter *d* affect the graph of $y = a(bx - c) + d$?**
  See margin.

EXPLAIN

A **linear function** is a relationship between an independent variable, usually x, and a dependent variable, usually y or $f(x)$, that generates a graph in the shape of a line.

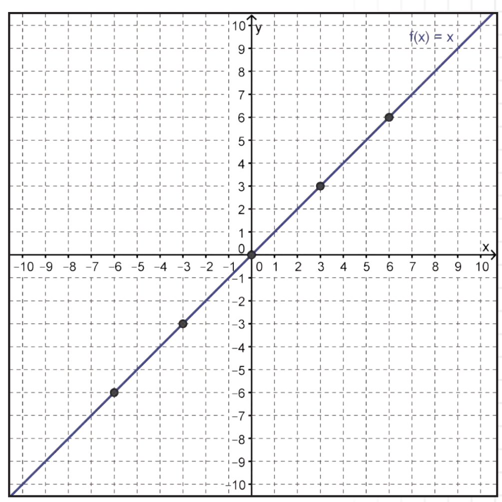
Watch Explain and You Try It Videos

or click here

You can represent a linear function using a table of values, graph, or symbols that you would use for an equation. The linear parent function, $f(x) = x$, is shown in the table, graph, and symbolic representation. The table contains a set of points from the domain and range of the linear parent function. These points are plotted on the graph.

$f(x) = x$

x	$f(x)$
-6	-6
-3	-3
0	0
3	3
6	6



The full family of linear functions is generated by applying transformations to the linear parent function. Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship.

10. *See page 139-A.*

REFLECT ANSWERS:

The parameter a generates a vertical stretch or compression or a reflection across the x-axis. If $|a| > 1$, then the graph is vertically stretched by a factor of a. If $0 < |a| < 1$, then the graph is vertically compressed by a factor of a. If $a < 0$, then the graph is reflected across the x-axis.

The parameter b generates a horizontal stretch or compression or a reflection across the y-axis. If $|b| > 1$, then the graph is horizontally compressed by a factor of $\frac{1}{|b|}$. If $0 < |b| < 1$, then the graph is horizontally stretched by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is reflected across the y-axis.

The parameter c works with the parameter b to generate a horizontal translation. If $c > 0$, then the graph is horizontally translated $|\frac{c}{b}|$ units to the right. If $c < 0$, then the graph is horizontally translated $|\frac{c}{b}|$ units to the left.

The parameter d generates a vertical translation. If $d > 0$, then the graph is vertically translated d units up. If $d < 0$, then the graph is vertically translated d units down.

QUESTIONING STRATEGIES

The general form of a linear function that can be written to compare it to higher-order functions is $f(x) = a(bx - c) + d$. Changes in b and c affect the value of the independent variable before they are applied to the function $y = a[x] + d$. Changes in a and d affect the value of the dependent variable after the linear function has been applied. Changes in a and b are multiplicative changes and changes in c and d are additive changes. Use questions such as these to help students make these connections.

- Which two parameters generate a similar type of change in the graph?
- How can you tell if a reflection is across the x-axis or the y-axis?
- How does the sign of a parameter influence the type of change in the graph?

INSTRUCTIONAL HINTS

Students will analyze multiple functions in Chapter 2. To help them organize and study the various changes and recognize patterns across all functions, have students create a graphic organizer of changes in *a*, *b*, *c*, and *d* for each function. Add to the graphic organizer in each section of the chapter.

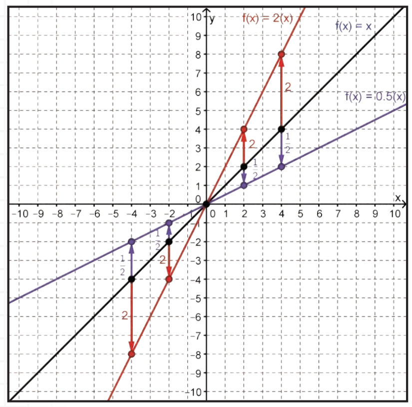
Draw students' attention to the arrows on the graphs. Correlate these arrows to the words "stretch" and "compress."

CHANGES IN *a*

The parameter *a* influences the vertical stretch or compression of the graph of the line.

> For a linear function, the general form is $f(x) = a(bx - c) + d$, where *a*, *b*, *c*, and *d* are real numbers.

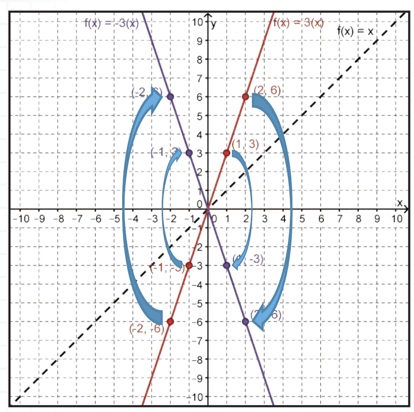
- If $|a| > 1$, then the range values (*y*-coordinates) of the original function are multiplied by a factor of *a* in order to vertically stretch the graph of the line.
- If $0 < |a| < 1$, then the range values (*y*-coordinates) of the original function are multiplied by a factor of *a* in order to vertically compress the graph of the line.



x	$f(x) = x$	$f(x) = 2(x)$	$f(x) = 0.5(x)$
-4	-4	-8	-2
-2	-2	-4	-1
0	0	0	0
2	2	4	1
4	4	8	2

× 2 × 0.5

The parameter *a* also affects the orientation of the line. If $a < 0$, then the line will be reflected across the *x*-axis.



x	$f(x) = 3(x)$	$f(x) = -3(x)$
-2	-6	6
-1	-3	3
0	0	0
1	3	-3
2	6	-6

× -1

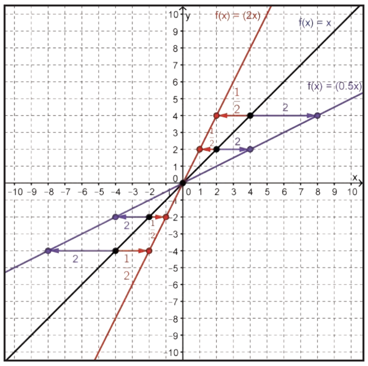
Domain (*x*) values are multiplied by −1 in order to generate the same range (*y*) value. This multiplication moves the points from Quadrant I to Quadrant IV and from Quadrant III to Quadrant II, generating a reflection across the *x*-axis.

140 CHAPTER 2: ANALYZING FUNCTIONS

CHANGES IN b

The parameter b influences the horizontal stretch or compression of the graph of the line.

- If $|b| > 1$, then the domain values (x-coordinates) of the original function are multiplied by a factor of $\frac{1}{|b|}$, which will be a multiplier that is less than 1, in order to horizontally compress the graph of the line.
- If $0 < |b| < 1$, then the domain values (x-coordinates) of the original function are multiplied by a factor of $\frac{1}{|b|}$, which will be a multiplier that is greater than 1, in order to horizontally stretch the graph of the line.



x	$f(x) = x$
-4	-4
-2	-2
0	0
2	2
4	4

x	$f(x) = (2x)$
-2	-4
-1	-2
0	0
1	2
2	4

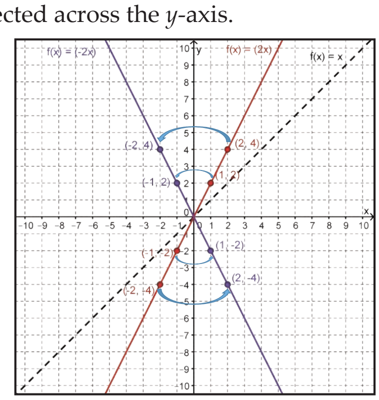
Domain (x) values are multiplied by $\frac{1}{2}$ in order to generate the same range (y) value. This multiplication results in a horizontal compression of the graph.

x	$f(x) = x$
-4	-4
-2	-2
0	0
2	2
4	4

x	$f(x) = (0.5x)$
-8	-4
-4	-2
0	0
4	2
8	4

Domain (x) values are multiplied by 2 in order to generate the same range (y) value. This multiplication results in a horizontal stretch of the graph.

The parameter b also affects the orientation of the line. If $b < 0$, then the line will be reflected across the y-axis.



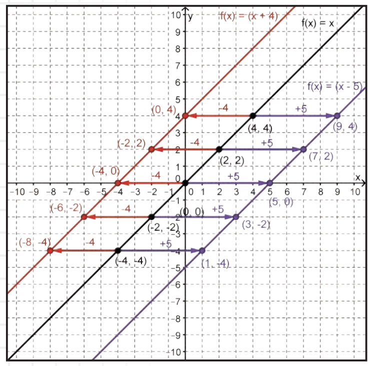
x	$f(x) = (2x)$	$f(x) = (-2x)$
-2	-4	4
-1	-2	2
0	0	0
1	2	-2
2	4	-4

Domain (x) values are multiplied by −1 in order to generate the same range (y) value. This multiplication moves the points from Quadrant I to Quadrant II and from Quadrant III to Quadrant IV, generating a reflection across the y-axis.

CHANGES IN c

The parameters b and c influence the horizontal translation of the graph of the line. Notice that in the general form, $f(x) = a(bx - c) + d$, the sign in front of c is negative. That means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation. For example, $f(x) = 2(3x - 5) + 1$ has $c = 5$ and $f(x) = 2(3x + 5) + 1$ has $c = -5$.

- If $c > 0$, then the graph of the line will translate $|\frac{c}{b}|$ units to the right.
- If $c < 0$, then the graph of the line will translate $|\frac{c}{b}|$ units to the left.



x	$f(x) = x$
-4	-4
-2	-2
0	0
2	2
4	4

x	$f(x) = (x - 5)$
1	-4
3	-2
5	0
7	2
9	4

Domain (x) values are increased by 5 in order to generate the same range (y) value. This addition results in a horizontal translation of the graph to the right.

x	$f(x) = x$
-4	-4
-2	-2
0	0
2	2
4	4

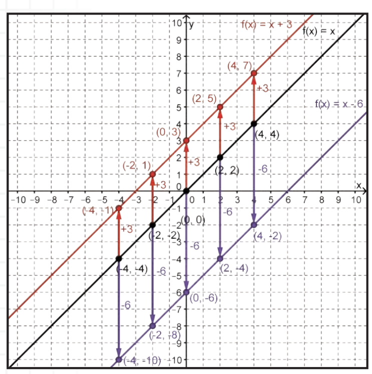
x	$f(x) = (x + 4)$
-8	-4
-6	-2
-4	0
-2	2
0	4

Domain (x) values are decreased by 4 in order to generate the same range (y) value. This subtraction (addition with a negative number) results in a horizontal translation of the graph to the left.

CHANGES IN d

The parameter d influences the vertical translation of the graph of the line.

- If $d > 0$, then the graph of the line will translate $|d|$ units up.
- If $d < 0$, then the graph of the line will translate $|d|$ units down.



x	$f(x) = x$	$f(x) = x - 6$	$f(x) = x + 3$
-4	-4	-10	-1
-2	-2	-8	1
0	0	-6	3
2	2	-4	5
4	4	-2	7

-6 +3

142 CHAPTER 2: ANALYZING FUNCTIONS

y-INTERCEPTS OF LINEAR FUNCTIONS

The **y-intercept** of a function is where the graph of the function intersects the y-axis. If the equation of the line is in slope-intercept form, $y = mx + b$, then the y-intercept is the point $(0, b)$. If the equation of the line is in general form, $y = a(bx - c) + d$, then the y-intercept can be calculated by substituting $x = 0$ into the general form.

$$\begin{aligned} y &= a(b(0) - c) + d \\ y &= a(0 - c) + d \\ y &= a(-c) + d \\ y &= -ac + d \end{aligned}$$

The ordered pair representing the y-intercept is $(0, -ac + d)$.

x-INTERCEPTS OF LINEAR FUNCTIONS

The **x-intercept** of a function is where the graph of the function intersects the x-axis. If the equation of the line is in general form, $y = a(bx - c) + d$, then the x-intercept can be calculated by substituting $y = 0$ into the general form.

$$\begin{aligned} 0 &= a(bx - c) + d \\ 0 &= abx - ac + d \\ 0 + ac - d &= abx - ac + ac + d - d \\ ac - d &= abx \\ \frac{ac - d}{ab} &= \frac{\cancel{ab}x}{\cancel{ab}} \\ \frac{ac - d}{ab} &= x \end{aligned}$$

The ordered pair representing the x-intercept is $(\frac{ac - d}{ab}, 0)$.

DOMAIN AND RANGE

The domain of a linear function is all real numbers. Using **set builder notation** the domain of a linear function is written as $\{x \mid x \in \mathbb{R}\}$. The range of a linear function is all real numbers. Using **set builder notation** the range of a linear function is written as $\{f(x) \mid f(x) \in \mathbb{R}.\}$

KEY ATTRIBUTES OF LINEAR FUNCTIONS

A linear function has several important key attributes:

- The domain of a linear function is all real numbers.
- The range of a linear function is all real numbers.
- There is not a maximum value or a minimum value for a linear function, as the line extends indefinitely in both directions.
- A linear function has one x-intercept at $(\frac{ac - d}{ab}, 0)$.
- A linear function has one y-intercept at $(0, -ac + d)$.

ADDITIONAL EXAMPLES

What transformations of the linear parent function, $f(x) = x$, will result in the graph of the linear functions below?

1. $h(x) = \frac{1}{2}(2x - 3) - 2$

The graph of h(x) is produced by transforming the linear parent function f(x) by vertically compressing its graph by a factor of one half, horizontally compressing its graph by a factor of one half, and translating its graph three halves of a unit right and two units down.

2. $k(x) = -5(\frac{1}{3}x + 3) + 4.5$

The graph of k(x) is produced by transforming the linear parent function f(x) by vertically stretching its graph by a factor of five, horizontally stretching its graph by a factor of three, reflecting its graph over the x-axis, and translating its graph nine units left and 4.5 units up.

3. $p(x) = \frac{2}{3}(-2x - 12) - \frac{3}{4}$

The graph of p(x) is produced by transforming the linear parent function f(x) by vertically compressing its graph by a factor of two thirds, horizontally compressing its graph by a factor of one half, reflecting its graph over the y-axis, and translating its graph six units to the left and three fourths of a unit down.

4. $r(x) = -(x - \frac{5}{2})$

The graph of r(x) is produced by transforming the linear parent function f(x) by reflecting its graph over the x-axis and translating its graph five halves units to the right.

EXAMPLE 1

What transformations of the linear parent function, $f(x) = x$, will result in the graph of the linear function $g(x) = -3(0.5x + 4) + \frac{2}{5}$?

STEP 1 **Rewrite the equation of $g(x)$ in general form to determine the values of the parameters a, b, c and d.**

$g(x) = -3(0.5x + 4) + \frac{2}{5}$

$g(x) = -3(0.5x - (-4)) + \frac{2}{5}$

Therefore, $a = -3$, $b = 0.5$, $c = -4$ and $d = \frac{2}{5}$.

STEP 2 **Use the values of the parameters to describe the transformations of the linear parent function $f(x)$ that are necessary to produce $g(x)$.**

$a = -3$, so $|a| > 1$. The range values (y-coordinates) of the linear parent function are multiplied by a factor of 3 in order to vertically stretch the graph of the line. Because the value of a is negative, the graph is also reflected over the x-axis.

$b = 0.5$, so $0 < b < 1$. The domain values (x-coordinates) of the linear parent function are multiplied by a factor of $\frac{1}{0.5} = 2$ in order to horizontally stretch the graph of the line.

$c = -4$, so $c < 0$. The graph of the linear parent function will translate $|\frac{-4}{0.5}| = 8$ units to the left.

$d = \frac{2}{5}$, so $d > 0$. The graph of the linear parent function will translate $|\frac{2}{5}| = \frac{2}{5}$ of a unit up.

The graph of g(x) is produced by transforming the linear parent function f(x) by vertically stretching its graph by a factor of three, horizontally stretching its graph by a factor of two, reflecting its graph over the x-axis, and translating its graph eight units to the left and two fifths of a unit up.

YOU TRY IT! #1

What transformations of the linear parent function, $f(x) = x$, will result in the graph of the linear function $h(x) = \frac{1}{4}(-6x - 5) + 1$?

See margin.

QUESTIONING STRATEGIES

In **ADDITIONAL EXAMPLE #3** above, $c > 1$ which should translate the graph of $p(x)$ to the right. However, $p(x)$ translated to the left. What caused this to happen?

In **ADDITIONAL EXAMPLE #4** above, certain terms appear to be missing. How does this affect your answer?

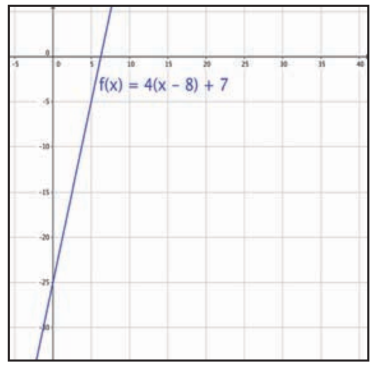
YOU TRY IT! #1 ANSWER:

The graph of h(x) is produced by transforming the linear parent function f(x) by vertically compressing its graph by a factor of one fourth, horizontally compressing its graph by a factor of one sixth, reflecting its graph over the y-axis, and translating its graph five sixths of a unit to the left and one unit up.

EXAMPLE 2

Identify the domain, range, x-intercept and y-intercept of the linear function described by the equation and graph shown below. Write the domain and range as inequalities and in set builder notation.

$f(x) = 4(x - 8) + 7$



STEP 1 **Determine the domain and range of $f(x)$.**

From the graph, the domain of $f(x)$ contains all real values of x. As an inequality, this is written as $-\infty < x < \infty$. In set builder notation, the domain is $\{x \mid x \in \mathbb{R}\}$.

Also analyzing the graph, the range of $f(x)$ contains all real values of $f(x)$. As an inequality, this is written as $-\infty < f(x) < \infty$. In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

STEP 2 **Determine the x-intercept of $f(x)$.**

x-intercepts of functions occur where the function value $f(x) = 0$.

$$\begin{aligned} 0 &= 4(x - 8) + 7 \\ 0 &= 4x - 32 + 7 \\ 0 &= 4x - 25 \\ 0 + 25 &= 4x - 25 + 25 \\ 25 &= 4x \\ \frac{25}{4} &= \frac{4x}{4} \\ 6.25 &= x \end{aligned}$$

The x-intercept of $f(x)$ is (6.25, 0).

STEP 3 **Determine the y-intercept of $f(x)$.**

y-intercepts of functions occur where the domain or input value $x = 0$.

$$\begin{aligned} f(0) &= 4(0 - 8) + 7 \\ f(0) &= 4(-8) + 7 \\ f(0) &= -32 + 7 \\ f(0) &= -25 \end{aligned}$$

The y-intercept of $f(x)$ is (0, – 25). The graph confirms this since the line intersects the y-axis at the point (0, – 25).

The domain of $f(x)$ is $-\infty < x < \infty$, or $\{x \mid x \in \mathbb{R}\}$. The range of $f(x)$ is $-\infty < f(x) < \infty$, or $\{f(x) \mid f(x) \in \mathbb{R}\}$. The x-intercept of $f(x)$ is (6.25, 0). The y-intercept of $f(x)$ is (0, – 25).

INSTRUCTIONAL HINT

Assist students in recalling set builder notation from previous mathematics courses.

In step 1 of Example 2, the domain is read as "the set of all x's such that x is all real numbers." The "double" R means all real numbers. The range is read as "the set of all $f(x)$ values such that $f(x)$ is all real numbers."

ADDITIONAL EXAMPLES

Identify the domain, range, x-intercept, and y-intercept of the linear functions described by the equations below. Write the domain and range as inequalities and in set builder notation.

1. $g(x) = -2(\frac{1}{2}x + 4) - 5$

The domain of $g(x)$ is $-\infty < x < \infty$, or $\{x \mid x \in \mathbb{R}\}$. The range of $g(x)$ is $-\infty < g(x) < \infty$, or $\{g(x) \mid (x) \in \mathbb{R}\}$. The x-intercept of $g(x)$ is (-13, 0). The y-intercept of $g(x)$ is (0, -13).

2. $h(x) = -(-6x - 12) + 3$

The domain of $h(x)$ is $-\infty < x < \infty$, or $\{x \mid x \in \mathbb{R}\}$. The range of $h(x)$ is $-\infty < h(x) < \infty$, or $\{h(x) \mid h(x) \in \mathbb{R}\}$. The x-intercept of $h(x)$ is $(-\frac{5}{2}, 0)$. The y-intercept of $h(x)$ is (0,15).

3. $j(x) = \frac{3}{4}(x - 16) - \frac{1}{4}$

The domain of $j(x)$ is $-\infty < x < \infty$, or $\{x \mid x \in \mathbb{R}\}$. The range of $j(x)$ is $-\infty < j(x) < \infty$, or $\{j(x) \mid j(x) \in \mathbb{R}\}$. The x-intercept of $j(x)$ is $(-12\frac{1}{4}, 0)$. The y-intercept of $j(x)$ is $(0, 16\frac{1}{3})$.

YOU TRY IT! #2 ANSWER:

Written as inequalities, the domain of $f(x)$ is $-\infty < x < \infty$, and the range of $f(x)$ is $-\infty < f(x) < \infty$. In set builder notation, the domain of $f(x)$ is $\{x \mid x \in \mathbb{R}\}$, and the range of $f(x)$ is $\{f(x) \mid f(x) \in \mathbb{R}\}$. The x-intercept is $(\frac{1}{3}, 0)$. The y-intercept is $(0, -1)$.

QUESTIONING STRATEGIES

Help students recall prior learning on writing domain and range as intervals.

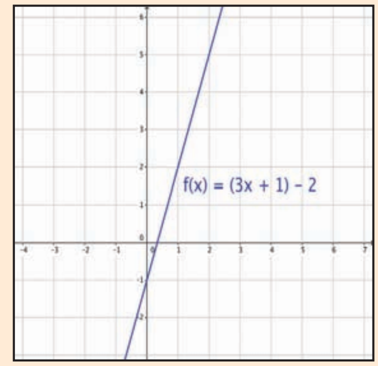
- What is the difference in parentheses and square brackets in interval notation?
- Why do infinity and negative infinity use parentheses rather than square brackets?

YOU TRY IT! #2

Identify the domain, range, x-intercept and y-intercept of the linear function described by the equation and graph shown below. Write the domain and range as inequalities and in set builder notation.

$f(x) = (3x + 1) - 2$

See margin.



EXAMPLE 3

Identify the domain, range, x-intercept and y-intercept of the linear function described by the equation and table shown below. Write the domain and range as intervals.

$f(x) = 2(\frac{1}{3}x) - 5$

x	$f(x)$
- 6	- 9
- 3	- 7
0	- 5
3	- 3
6	- 1

STEP 1 **Determine the domain and range of $f(x)$.**

Since $f(x)$ is a linear function, its domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both $(-\infty, \infty)$.

STEP 2 **Determine the x-intercept of $f(x)$.**

x-intercepts of functions occur where the output or function value $f(x) = 0$.

$$0 = 2(\tfrac{1}{3}x) - 5$$
$$0 = \tfrac{2}{3}x - 5$$
$$0 + 5 = \tfrac{2}{3}x - 5 + 5$$
$$5 = \tfrac{2}{3}x$$
$$\tfrac{3}{2}(5) = \tfrac{3}{2}(\tfrac{2}{3}x)$$
$$7.5 = x$$

The x-intercept of $f(x)$ is $(7.5, 0)$.

ADDITIONAL EXAMPLES

Identify the domain, range, x-intercept, and y-intercept of the linear functions described by the equations and tables below. Write the domain and range as intervals.

1. $p(x) = -3(2x - 0.5) + 2.5$

x	y
-2	16
0	4
2	-8
4	-20
6	-32

The domain of $p(x)$ is $-\infty < x < \infty$. The range of $p(x)$ is $-\infty < p(x) < \infty$. The x-intercept of $p(x)$ is $(\frac{2}{3}, 0)$. The y-intercept of $p(x)$ is $(0, 4)$.

2. $q(x) = \frac{1}{3}(-\frac{3}{2}x + 12) - 5$

x	y
-6	2
-5	1.5
-4	1
-3	0.5
-2	0

The domain of $q(x)$ is $-\infty < x < \infty$. The range of $q(x)$ is $-\infty < q(x) < \infty$. The x-intercept of $q(x)$ is $(-2, 0)$. The y-intercept of $q(x)$ is $(0, -1)$.

STEP 3 **Determine the y-intercept of $f(x)$.**

y-intercepts of functions occur where the domain or input value $x = 0$. From the table, the y-intercept of $f(x)$ is $(0, -5)$.

The domain of $f(x)$ is $(-\infty, \infty)$. The range of $f(x)$ is $(-\infty, \infty)$. The x-intercept of $f(x)$ is $(7.5, 0)$. The y-intercept of $f(x)$ is $(0, -5)$.

YOU TRY IT! #3

Identify the domain, range, x-intercept and y-intercept of the linear function described by the equation and table shown below. Write the domain and range as intervals.

$f(x) = \frac{1}{2}(5x - 1)$

See margin.

x	y
– 3	– 8
– 1	– 3
1	2
3	7
5	12

YOU TRY IT! #3 ANSWER:

The domain of $f(x)$ is $(-\infty, \infty)$. The range of $f(x)$ is $(-\infty, \infty)$. The x-intercept is $(\frac{1}{5}, 0)$. The y-intercept is $(0, -\frac{1}{2})$.

PRACTICE/HOMEWORK

For questions 1 - 8, describe the transformation of the linear parent function, $f(x) = x$ that will result in the graph of the linear function given.

1. $h(x) = (4x - 1)$
See margin.

2. $g(x) = -2(x) + 5$
See margin.

3. $h(x) = 3(x + 2)$
See margin.

4. $g(x) = (-\frac{1}{2}x + 3) + 7$
See margin.

5. $h(x) = -\frac{3}{4}(x - 8) + 2$
See margin.

6. $g(x) = \frac{2}{3}(6x + 1) - 3$
See margin.

7. $h(x) = -4(\frac{1}{2}x - 3) + 4$
See margin.

8. $g(x) = -(-8x + 9) - 6$
See margin.

1. *The graph of $h(x)$ is produced by transforming the linear parent function $f(x)$ by horizontally compressing its graph by a factor of one fourth and translating its graph $\frac{1}{4}$ unit to the right.*

2. *The graph of $g(x)$ is produced by transforming the linear parent function $f(x)$ by vertically stretching its graph by a factor of two, reflecting its graph over the x-axis, and translating its graph five units up.*

3. *The graph of $h(x)$ is produced by transforming the linear parent function $f(x)$ by vertically stretching its graph by a factor of three and translating its graph two units to the left.*

4. *The graph of $g(x)$ is produced by transforming the linear parent function $f(x)$ by horizontally stretching its graph by a factor of two, reflecting its graph over the y-axis, and translating its graph six units to the right and seven units up.*

5. *The graph of $h(x)$ is produced by transforming the linear parent function $f(x)$ by vertically compressing its graph by a factor of three fourths, reflecting its graph over the x-axis, and translating its graph eight units to the right and two units up.*

6. *The graph of $g(x)$ is produced by transforming the linear parent function $f(x)$ by vertically compressing its graph by a factor of two thirds, horizontally compressing its graph by a factor of one sixth, and translating its graph $\frac{1}{6}$ unit to the left and three units down.*

7. *The graph of $h(x)$ is produced by transforming the linear parent function $f(x)$ by vertically stretching its graph by a factor of four, horizontally stretching its graph by a factor of two, reflecting its graph over the x-axis, and translating its graph six units to the right and four units up.*

8. *The graph of $g(x)$ is produced by transforming the linear parent function $f(x)$ by horizontally compressing its graph by a factor of one eighth, reflecting its graph over the x-axis, reflecting its graph over the y-axis, and translating its graph $\frac{9}{8}$ units to the right and six units down.*

9. *The domain of f(x) contains all real values of x. As an inequality, this is written as* $-\infty < x < \infty$.

The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$.

The x-intercept of f(x) is (-1.5, 0).

The y-intercept of f(x) is (0, 3).

10. *The domain of f(x) contains all real values of x. As an inequality, this is written as* $-\infty < x < \infty$.

The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$.

The x-intercept of f(x) is (-15, 0).

The y-intercept of f(x) is (0, -5).

11. *The domain of f(x) contains all real values of x. As an inequality, this is written as* $-\infty < x < \infty$.

The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$.

The x-intercept of f(x) is (6, 0).

The y-intercept of f(x) is (0, 3).

12. *The domain of f(x) contains all real values of x. As an inequality, this is written as* $-\infty < x < \infty$.

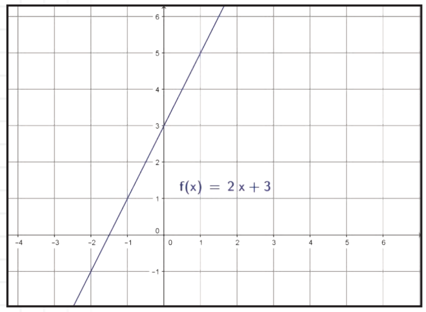
The range of f(x) contains all real values of f(x). As an inequality, this is written as $-\infty < f(x) < \infty$.

The x-intercept of f(x) is (8, 0).

The y-intercept of f(x) is (0, 8).

For questions 9 - 12, identify the domain, range, x-intercept, and y-intercept of the linear function described by the equation and the graph. Write the domain and range as inequalities.

9. $f(x) = (2x + 3)$



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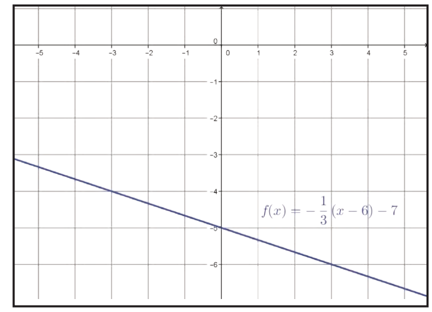
10. $f(x) = -\frac{1}{3}(x - 6) - 7$



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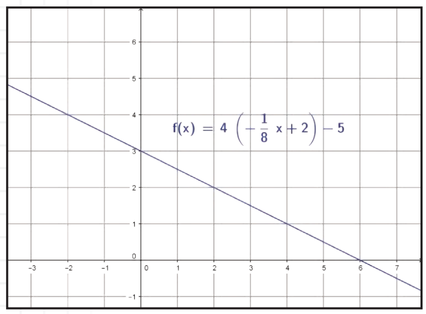
11. $f(x) = 4(-\frac{1}{8}x + 2) - 5$



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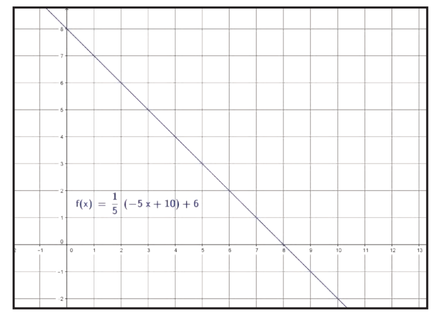
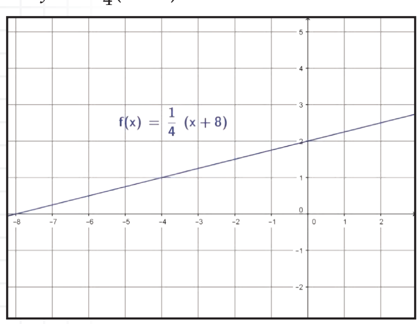
12. $f(x) = \frac{1}{5}(-5x + 10) + 6$



See margin.

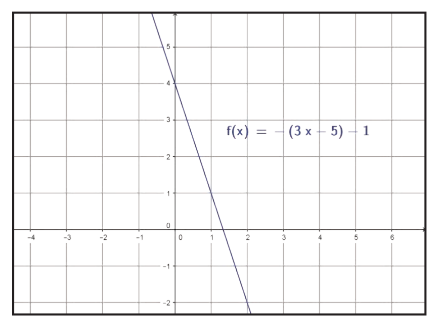
For questions 13 - 16, identify the domain, range, x-intercept, and y-intercept of the linear function described by the equation and the graph. Write the domain and range in set builder notation.

13. $f(x) = \frac{1}{4}(x + 8)$



See margin.

14. $f(x) = -(3x - 5) - 1$



See margin.

13. *The domain of f(x) contains all real values of x. In set builder notation, the domain is* $\{x \mid x \in \mathbb{R}\}$.

The range of f(x) contains all real values of f(x). In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is (-8, 0).

The y-intercept of f(x) is (0, 2).

14. *The domain of f(x) contains all real values of x. In set builder notation, the domain is* $\{x \mid x \in \mathbb{R}\}$.

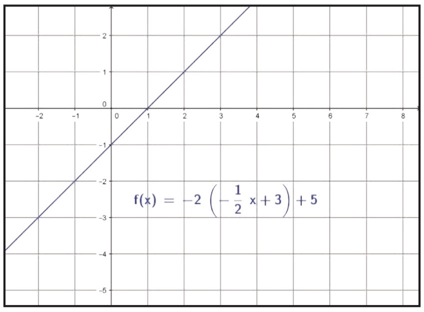
The range of f(x) contains all real values of f(x). In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is $(\frac{4}{3}, 0)$.

The y-intercept of f(x) is (0, 4).

148 CHAPTER 2: ANALYZING FUNCTIONS

15. $f(x) = -2(-\frac{1}{2}x + 3) + 5$



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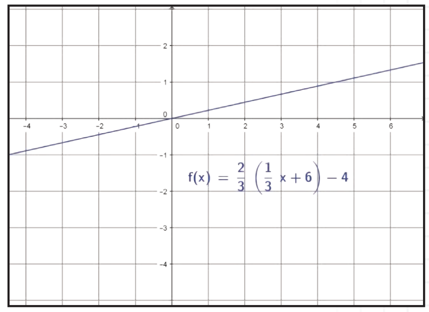
16. $f(x) = \frac{2}{3}(\frac{1}{3}x + 6) - 4$



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For questions 17 - 20, identify the domain, range, x-intercept, and y-intercept of the linear function described by the equation and the table. Write the domain and range as intervals.

17. $f(x) = -\frac{2}{5}(x + 8)$

x	$f(x)$
-6	-0.8
-3	-2
2	-4
4	-4.8
7	-6

See margin.

18. $f(x) = \frac{1}{2}(-6x - 3)$

x	$f(x)$
-4	10.5
-2	4.5
1	-4.5
3	-10.5
5	-16.5

See margin.

19. $f(x) = -3(2x + 5) + 9$

x	$f(x)$
-5	24
-3	12
1	-12
2	-18
4	-30

See margin.

20. $f(x) = -\frac{1}{3}(-\frac{1}{4}x - 9) - 5$

x	$f(x)$
-12	-3
-6	-2.5
-3	-2.25
3	-1.75
9	-1.25

See margin.

15. *The domain of f(x) contains all real values of x. In set builder notation, the domain is* $\{x \mid x \in \mathbb{R}\}$.

The range of f(x) contains all real values of f(x). In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is (1, 0).

The y-intercept of f(x) is (0, -1).

16. *The domain of f(x) contains all real values of x. In set builder notation, the domain is* $\{x \mid x \in \mathbb{R}\}$.

The range of f(x) contains all real values of f(x). In set builder notation, the range is $\{f(x) \mid f(x) \in \mathbb{R}\}$.

The x-intercept of f(x) is (0, 0).

The y-intercept of f(x) is (0, 0).

17. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both* $(-\infty, \infty)$.

The x-intercept of f(x) is (-8, 0).

The y-intercept of f(x) is (0, -3.2).

18. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both* $(-\infty, \infty)$.

The x-intercept of f(x) is (-0.5, 0).

The y-intercept of f(x) is (0, -1.5).

19. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both* $(-\infty, \infty)$.

The x-intercept of f(x) is (-1, 0).

The y-intercept of f(x) is (0, -6).

20. *The domain and range include all real numbers. Real numbers are infinite. Written as intervals, the domain and the range are both* $(-\infty, \infty)$.

The x-intercept of f(x) is (24, 0).

The y-intercept of f(x) is (0, -2).