Study Guide and Intervention
Transforming and Analyzing Cubic Functions

Example **Describe the transformation.**

What transformations of the cubic parent function, $f(x) = x^3$, will result in the graph of the cubic function $g(x) = -2(x + 3)3 + 4.5$?

Solution

Step 1 Rewrite the equation of g(x) in general form to determine the values of the parameters a, b, c, and d.

$$g(x) = -2(x + 3)3 + 4.5$$
$$g(x) = -2(x - (-3))3 + 4.5$$
Therefore, a = -2, b = 1, c = -3, and d = 4.5

a = -2, so |a| > 1. The range values (y-coordinates) of the cubic parent function are multiplied by a factor of 2 in order to vertically stretch the graph

since a < 0, the graph will be reflected across the x-axis

b = 1, so there is no change.

Step 2 Use the values of the parameters to describe the transformations of the cubic parent function f(x) that are necessary to produce g(x).

c = -3, so c < 0. The graph of the cubic parent function will translate $\left|\dfrac{-3}{1}\right|$ = 3 units to the left

d = 4.5, so the graph will translate 4.5 units up

Exercises

For questions 1-4, describe the transformation of the cubic parent function, $f(x) = x^3$ that will result in the graph of the cubic function given.

1. $h(x) = (2x - 1)^3$

2. $h(x) = 2(x + 2)^3$

3. $h(x) = -\dfrac{3}{4}(x - 6)^3 + 3$

4. $h(x) = -3\left(\dfrac{1}{4}x - 1\right)^3 + 5$

Study Guide and Intervention
Transforming and Analyzing Cubic Functions (cont.)

Example Identify the domain, range, relative minimum, relative maximum, x-intercept and y-intercept of the cubic function described by the equation shown below. Write the domain and range as intervals, and in set builder notation.

$$f(x) = (x + 1)(x - 3)(2x - 1)$$

Solution

Step 1 Determine the domain and range of f(x).

Since this is a cubic function, the domain and range are *all real numbers*; the range is found using the equation.

As an interval

Domain: $(-\infty, \infty)$ Range: $[-2, \infty)$

Set builder notation

Domain: $\{x | x \in \mathbb{R}\}$ Range: $\{y | y \geq -2\}$

Step 2 Determine the x-intercept of f(x).

Use your calculator to find the x-intercepts

(-1, 0); (3, 0); (.5, 0)

Step 3 Determine the y-intercept of f(x).

The y-intercept is found when x = 0, or can be found using $(0, -ac^3 + d)$

$(x + 1)(x - 3)(2x - 1)$
$(0 + 1)(0 - 3)(2*0 - 1)$
$(1)(-3)(-1) = 3$
(0, 3)

Step 4 Determine the relative minimum and maximum

Use your calculator to find the relative minimum and maximum

Min: (2, -9)
Max: (-.33, 3.7)

Exercises

For questions 5-7, identify the domain, range, x-intercept and y-intercept of the cubic function described by the equation shown below. Write the domain and range as intervals, and in set builder notation.

5. $f(x) = (3x + 2)^3$

6. $f(x) = 2(-\frac{1}{4}x + 2)^3 - 3$

7. $f(x) = 7(-3x - 12)^3 - 14$

For questions 8-9, identify the domain, range, relative minimum, relative maximum, x-intercept and y-intercept of the cubic function described by the equation shown below. Write the domain and range as intervals, and in set builder notation.

8. $f(x) = (2x + 5)(x - 3)(4x - 1)$

9. $f(x) = (x - 4)(x + 2)(3x + 1)$