

# Study Guide and Intervention

## Transforming and Analyzing Quadratic Functions

**Example** Describe the transformation.

What transformations of the quadratic parent function,  $f(x) = x^2$ , will result in the graph of the quadratic function  $g(x) = 2(x + 3)^2$ ?

**Solution**

**Step 1** Rewrite the equation of  $g(x)$  in general form to determine the values of the parameters  $a$ ,  $b$ ,  $c$ , and  $d$ .

$$g(x) = 2(x + 3)^2$$

$$g(x) = 2(x - (-3))^2$$

Therefore,  $a = 2$ ,  $b = 1$ ,  $c = -3$ , and  $d = 0$

$b = 1$ , so there is no change.

$c = -3$ , so  $c < 0$ . The graph of the quadratic parent function will translate  $\left| \frac{-3}{1} \right| = 3$  units to the left

$d = 0$ , so there is no change

**Step 2** Use the values of the parameters to describe the transformations of the quadratic parent function  $f(x)$  that are necessary to produce  $g(x)$ .

$a = 2$ , so  $|a| > 1$ . The range values (y-coordinates) of the quadratic parent function are multiplied by a factor of  $2$  in order to **vertically stretch the graph**

### Exercises

For questions 1-4, describe the transformation of the quadratic parent function,  $f(x) = x^2$  that will result in the graph of the quadratic function given.

1.  $h(x) = \frac{-1}{4}(x)^2 + 5$

2.  $h(x) = \left(\frac{1}{2}x\right)^2 - 1$

3.  $h(x) = \frac{1}{3}(2x - 5)^2 - 4$

4.  $h(x) = -2(3x + 10)^2 + 3$

# Study Guide and Intervention

## Transforming and Analyzing Quadratic Functions (cont.)

**Example** Identify the domain, range, x-intercept, y-intercept and vertex of the quadratic function described by the equation shown below. Write the domain and range as inequalities, as intervals, and in set builder notation.

$$f(x) = (4x - 1)^2 - 2$$

**Solution**

**Step 1** Determine the domain and range of  $f(x)$ .

Since this is a quadratic function, the domain is *all real numbers*; the range is found using the equation. Since  $a$  is positive, the graph opens up. Since  $d = -2$ , the range contains all real values that are greater than or equal to  $-2$ .

As an inequality

$$\text{Domain: } -\infty < x < \infty, \text{ Range: } -2 \leq y < \infty$$

As an interval

$$\text{Domain: } (-\infty, \infty) \text{ Range: } [-2, \infty)$$

Set builder notation

$$\text{Domain: } \{x|x \in \mathbb{R}\} \text{ Range: } \{y|y \geq -2\}$$

**Step 2** Determine the x-intercept of  $f(x)$ .

$$\text{x-intercepts are located at } \left(\frac{c \pm \sqrt{(-d)}}{b}, 0\right)$$

$$\left(\frac{c \pm \sqrt{(-d)}}{b}, 0\right)$$

$$\left(\frac{1 \pm \sqrt{(-(-2))}}{4}, 0\right)$$

$$\left(\frac{1 \pm \sqrt{2}}{4}, 0\right)$$

$$\left(\frac{1+\sqrt{2}}{4}, 0\right), \left(\frac{1-\sqrt{2}}{4}, 0\right)$$

**Step 3** Determine the y-intercept of  $f(x)$ .

y-intercepts is located at  $(0, ac^2 + d)$

$$(0, ac^2 + d)$$

$$(0, 1 \cdot 1^2 - 2)$$

$$(0, 1 - 2)$$

$$(0, -1)$$

**Step 4** Determine the vertex of the parabola

The vertex is located at  $\left(\frac{c}{b}, d\right)$

$$\left(\frac{c}{b}, d\right)$$

$$\left(\frac{1}{4}, -2\right)$$

### Exercises

For questions 5-10, identify the domain, range, x-intercept, y-intercept and vertex of the quadratic function described by the equation shown below. Write the domain and range as inequalities, as intervals, and in set builder notation.

5.  $f(x) = -\frac{1}{2}x - 3)^2 - 1$

6.  $f(x) = 2(x + 1)^2 - 8$

7.  $f(x) = (x - 2)^2 - 9$

8.  $f(x) = -3(4x)^2 + 7$

9.  $f(x) = \frac{-1}{2}(-10x + 4)^2 + 6$

10.  $f(x) = 6(2x - 10)^2 - 3$