Writing Cubic Functions Δ^2y =

For questions 1-6, determine whether the set of data represents a linear, exponential, quadratic, or cubic function.

2.

x	y
0	-1.25
1	-1
2	-0.75
3	-0.5
4	-0.25
5	0

SOLUTION:

$\Delta y = .25$

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$\Delta y = .25$

$\Delta y = .25$

The first finite differences are all .25, so the data represents a linear function

ANSWER:

Linear function

4.

x	$f(x)$
-1	-2
0	-8
1	-2
2	16
3	46
4	88

SOLUTION:

$\Delta y = -6$ $\Delta^2y = 12$

$\Delta y = 6$ $\Delta^2y = 12$

$\Delta y = 18$ $\Delta^2y = 12$

$\Delta y = 30$ $\Delta^2y = 12$

$\Delta y = 42$ $\Delta^2y = 12$

The second finite differences are all 12, so the data represents a quadratic function

ANSWER:

Quadratic function

6.

x	y
1	40
2	38
3	36
4	34
5	32
6	30

SOLUTION:

$\Delta y = -2$

$\Delta y = -2$

$\Delta y = -2$

$\Delta y = -2$

$\Delta y = -2$

The first finite differences are all -2, so the data represents a linear function

ANSWER:

Linear function

For questions 8-10, determine if the given relationship is a cubic function. If it is, write a function relating the variables.

8.

x	y
-1	8
0	5
1	6
2	11
3	20
4	33

SOLUTION:

$\Delta y = -3$ $\Delta^2 y = 4$

$\Delta y = 1$ $\Delta^2 y = 4$

$\Delta y = 5$ $\Delta^2 y = 4$

$\Delta y = 9$ $\Delta^2 y = 4$

$\Delta y = 13$

The second finite differences are all 4, so the data represents a quadratic function

ANSWER:

Not a cubic function

10.

x	y
0	-7
1	-5
2	9
3	47
4	121
5	243

SOLUTION:

$\Delta y = 2$ $\Delta^2 y = 12$ $\Delta^3 y = 12$

$\Delta y = 14$ $\Delta^2 y = 24$ $\Delta^3 y = 12$

$\Delta y = 38$ $\Delta^2 y = 36$ $\Delta^3 y = 12$

$\Delta y = 74$ $\Delta^2 y = 48$

$\Delta y = 122$

The third finite differences are all 12, so the data represents a quadratic function

$\Delta^3 y = 12$, $6a = 12$; $a = 2$

$\Delta^2 y = 12$, $6a + 2b = 12$; $12 + 2b = 12$; $b = 0$

$\Delta y = 2$, $a + b + c = 2$; $2 + 0 + c = 2$; $c = 0$

y-int = $d = -7$

ANSWER:

$y = 2x^2 - 7$

For questions 11-16, the data sets shown in the tables represent cubic functions. Write a cubic function for the values in the table.

12.

x	$f(x)$
0	-5
1	-4.8
2	-3.4
3	0.4
4	7.8
5	20

SOLUTION:

$\Delta y = 0.2$ $\Delta^2 y = 1.2$ $\Delta^3 y = 1.2$

$\Delta y = 1.4$ $\Delta^2 y = 2.4$ $\Delta^3 y = 1.2$

$\Delta y = 3.8$ $\Delta^2 y = 3.6$ $\Delta^3 y = 1.2$

$\Delta y = 7.4$ $\Delta^2 y = 4.8$

$\Delta y = 12.2$

The third finite differences are all 1.2, so the data represents a cubic function

$\Delta^3 y = 1.2$, $6a = 1.2$; $a = 0.2$

$\Delta^2 y = 1.2$, $6a + 2b = 1.2$; $1.2 + 2b = 1.2$; $b = 0$

$\Delta y = 0.2$, $a + b + c = 0.2$; $0.2 + 0 + c = 0.2$; $c = 0$

y-int = $d = -5$

ANSWER:

$y = 0.2x^2 - 5$

14.

x	y
0	0
1	-4
2	0
3	18
4	56
5	120

SOLUTION:

$\Delta y = -4$ $\Delta^2 y = 8$ $\Delta^3 y = 6$

$\Delta y = 4$ $\Delta^2 y = 14$ $\Delta^3 y = 6$

$\Delta y = 18$ $\Delta^2 y = 20$ $\Delta^3 y = 6$

$\Delta y = 38$ $\Delta^2 y = 26$

$\Delta y = 64$

The third finite differences are all 6, so the data represents a cubic function

$\Delta^3 y = 6$, $6a = 6$; $a = 1$

$\Delta^2 y = 8$, $6a + 2b = 8$; $6 + 2b = 8$; $2b = 2$; $b = 1$

$\Delta y = -4$, $a + b + c = -4$; $1 + 1 + c = -4$; $c = -6$

y-int = $d = 0$

ANSWER:

$y = x^3 + x^2 - 6x$

16.

x	y
0	0
1	-27
2	-60
3	-63
4	0
5	165

SOLUTION:

$\Delta y = -27$ $\Delta^2 y = -6$ $\Delta^3 y = 36$

$\Delta y = -33$ $\Delta^2 y = 30$ $\Delta^3 y = 36$

$\Delta y = -3$ $\Delta^2 y = 66$ $\Delta^3 y = 36$

$\Delta y = 63$ $\Delta^2 y = 102$

$\Delta y = 165$

The third finite differences are all 36, so the data represents a cubic function

$\Delta^3 y = 36$, $6a = 36$; $a = 6$

$\Delta^2 y = -6$, $6a + 2b = -6$; $36 + 2b = -6$; $2b = -42$; $b = -21$

$\Delta y = -27$, $a + b + c = -27$; $6 - 21 + c = -27$; $-15 + c = -27$

$c = -12$

y-int = $d = 0$

ANSWER:

$y = 6x^3 - 21x^2 - 12x$

A local mail service charges different rates, based on the weight of the packages mailed. A sample of their prices is shown in the table below.

WEIGHT OF PACKAGE, w (POUNDS)	PRICE TO MAIL PACKAGE, p ($)
0	0
1	3.45
2	6.60
3	10.65
4	16.80
5	26.25

20. Use a cubic function to determine the cost to mail a 6-pound package.

SOLUTION:

$\Delta y = 3.45$ $\Delta^2 y = -0.3$ $\Delta^3 y = 1.2$

$\Delta y = 3.15$ $\Delta^2 y = 0.9$ $\Delta^3 y = 1.2$

$\Delta y = 4.05$ $\Delta^2 y = 2.1$ $\Delta^3 y = 1.2$

$\Delta y = 6.15$ $\Delta^2 y = 3.3$

$\Delta y = 9.45$

The third finite differences are all 1.2, so the data represents a cubic function

$\Delta^3 y = 1.2$, $6a = 1.2$; $a = 0.2$

$\Delta^2 y = -0.3$, $6a + 2b = -0.3$; $1.2 + 2b = -0.3$; $2b = -1.5$;

$b = -0.75$

$\Delta y = 3.45$, $a + b + c = 3.45$; $0.2 – 0.75 + c = 3.45$;

$-0.55 + c = 3.45$;

$c = 4$

y-int = $d = 0$

$y = 0.2x^3 – 0.75x^2 + 4x$

Input 6 for x

$y = 0.2(6)^3 – 0.75(6)^2 + 4(6)$

ANSWER:

y = $40.20