Writing Quadratic Functions $\Delta^2 y =$

For questions 1-8, use finite differences and mental math, as appropriate, to determine if the data sets shown in the tables below represent a linear, exponential, quadratic, or other type of function.

2.

x	$y = f(x)$
1	5
2	11
3	17
4	23
5	29

SOLUTION:

$\Delta y = 11 - 5 = 6$

$\Delta y = 17 - 11 = 6$

$\Delta y = 23 - 17 = 6$

$\Delta y = 29 - 23 = 6$

The first finite differences are all 6, so the data represents a linear function

ANSWER:

Linear function

4.

x	$y = f(x)$
1	5
2	14
3	29
4	50
5	77

SOLUTION:

$\Delta y = 14 - 5 = 9$ $\quad \Delta^2 y = 15 - 9 = 6$

$\Delta y = 29 - 14 = 15$ $\quad \Delta^2 y = 21 - 15 = 6$

$\Delta y = 50 - 29 = 21$ $\quad \Delta^2 y = 27 - 21 = 6$

$\Delta y = 77 - 50 = 27$

The second finite differences are all 6, so the data represents a quadratic function

ANSWER:

Quadratic function

6.

x	$y = f(x)$
1	5
2	8
3	13
4	20
5	29

SOLUTION:

$\Delta y = 8 - 5 = 3$ $\quad \Delta^2 y = 5 - 3 = 2$

$\Delta y = 13 - 8 = 5$ $\quad \Delta^2 y = 7 - 5 = 2$

$\Delta y = 20 - 13 = 7$ $\quad \Delta^2 y = 9 - 7 = 2$

$\Delta y = 29 - 20 = 9$

The second finite differences are all 2, so the data represents a quadratic function

ANSWER:

Quadratic function

8.

x	$y = f(x)$
1	5
2	9
3	13
4	17
5	21

SOLUTION:

$\Delta y = 9 - 5 = 4$

$\Delta y = 13 - 9 = 4$

$\Delta y = 17 - 13 = 4$

$\Delta y = 21 - 17 = 4$

The first finite differences are all 4, so the data represents a linear function

ANSWER:

Linear function

For questions 9-12, the data sets shown in the tables represent quadratic functions. Use finite differences to determine the values of a, b, and c and then write the function in standard form.

10.

x	$y = f(x)$
0	3
1	6
2	13
3	24

SOLUTION:

$\Delta y = 6 - 3 = 3$ $\Delta^2 y = 7 - 3 = 4$

$\Delta y = 13 - 6 = 7$ $\Delta^2 y = 11 - 7 = 4$

$\Delta y = 24 - 13 = 11$

The second finite differences are all 4, so the data represents a quadratic function

$\Delta^2 y = 2a = 4$; $a = 2$

$\Delta y = a + b = 3$; $2 + b = 3$; $b = 1$

When $x = 0$, $y = 3$; $c = 3$

ANSWER:

$y = 2x^2 + x + 3$

12.

x	$y = f(x)$
0	–6
1	–1
2	14
3	39

SOLUTION:

$\Delta y = -1 - (-6) = 5$ $\Delta^2 y = 15 - 5 = 10$

$\Delta y = 14 - (-1) = 15$ $\Delta^2 y = 25 - 15 = 10$

$\Delta y = 39 - 14 = 25$

The second finite differences are all 10, so the data represents a quadratic function

$\Delta^2 y = 2a = 10$; $a = 5$

$\Delta y = a + b = 5$; $5 + b = 5$; $b = 0$

When $x = 0$, $y = -6$; $c = -6$

ANSWER:

$y = 5x^2 - 6$

For questions 13-16, the data sets shown in the tables represent quadratic functions. Use finite differences to determine f(0), the values of a, b, and c and then write the function in standard form.

14.

x	$y = f(x)$
0	?
1	3
2	16
3	41
4	78

SOLUTION:

$\Delta y = 16 - 3 = 13$ $\Delta^2 y = 25 - 13 = 12$

$\Delta y = 41 - 16 = 25$ $\Delta^2 y = 37 - 25 = 12$

$\Delta y = 78 - 41 = 37$

The second finite differences are all 12, so the data represents a quadratic function

Since the second finite differences are all 12, the previous difference must also be 12.

$\Delta^2y = 13 – x = 12; x = 1$

$3 – 1 = 2; f(0) = 2$

$\Delta^2y = 2a = 12; a = 6$

$\Delta y = a + b = 1; 6 + b = 1; b = -5$

When $x = 0, y = 2; c = 2$

ANSWER:

$f(0) = 2$

$a = 6, b = -5, c = 2$

$y = 6x^2 – 5x + 2$

16.

x	$y = f(x)$
0	?
1	7
2	22
3	45
4	76

SOLUTION:

$\Delta y = 22 - 7 = 15$ $\Delta^2y = 23 – 15 = 8$

$\Delta y = 45 - 22 = 23$ $\Delta^2y = 31 – 23 = 8$

$\Delta y = 76 - 45 = 31$

The second finite differences are all 8, so the data represents a quadratic function

Since the second finite differences are all 8, the previous difference must also be 8.

$\Delta^2y = 15 – x = 8; x = 7$

$7 – 7 = 0; f(0) = 0$

$\Delta^2y = 2a = 8; a = 4$

$\Delta y = a + b = 7; 4 + b = 7; b = 3$

When $x = 0, y = 0; c = 0$

ANSWER:

$f(0) = 0$

$a = 4, b = 3, c = 0$

$y = 4x^2 +3x$