Modeling with Exponential Functions

1.5

FOCUSING QUESTION How can you use common ratios to construct an exponential model for a data set?

LEARNING OUTCOMES

- I can use finite differences or common ratios to classify a function as either linear or exponential when I am given a table of values.
- I can use common ratios to write an exponential function that describes a data set.
- I can apply mathematics to problems that I see in everyday life, in society, and in the workplace.

ENGAGE

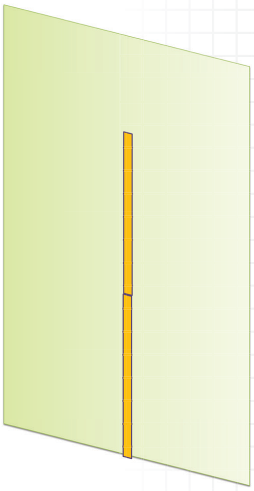
DeAnna dropped a basketball and let it bounce several times. What would a graph of the height of the basketball versus time look like?

See margin.

EXPLORE

According to international basketball guidelines, a basketball should be inflated such that when dropped from a height of 1.8 meters, the ball should bounce and rebound to a height of at least 1.2 meters but no more than 1.4 meters.

Bounce height is an important aspect to making sure that a ball used in any sport is properly inflated or is not worn out.



DIRECTIONS

- Tape two meter sticks to the wall and use them to measure the bounce height, or the height to which the ball bounces, when dropped from a given height.
- Begin with 180 centimeters. Select a spot on the ball, such as the highest point on the top of the ball, to use as a consistent reference point. Drop the ball from this height and record the height of the first bounce.
- Repeat for two more trials and calculate the average bounce height for a drop from 180 centimeters.

TEKS

AR.2B Classify a function as linear, quadratic, cubic, and exponential when a function is represented tabularly using finite differences or common ratios as appropriate.

AR.2D Determine a function that models real-world data and mathematical contexts using finite differences such as the age of a tree and its circumference, figurative numbers, average velocity, and average acceleration.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

5B Write using newly acquired basic vocabulary and content-based grade-level vocabulary.

VOCABULARY

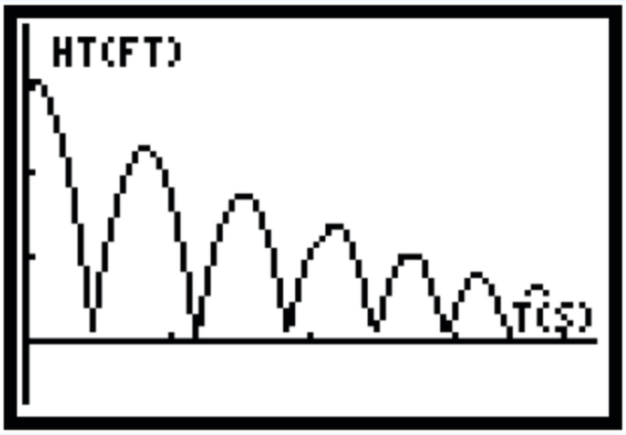
exponential function, common ratios, exponential decay, exponential growth

MATERIALS

- 1 ball for each student group (variety of balls, such as soccer balls, basketballs, tennis balls, and racquetballs, keeping in mind that not all balls will bounce on carpet)
- 2 meter sticks per student group
- masking tape

ENAGE ANSWER:

Possible sketch



STRATEGIES FOR SUCCESS

It is always best to have students collect data on their own. However, if it is not possible to collect data for this activity, provide students with the sample data so that they can use it to answer the questions with this activity.

Provide different student groups with different balls; e.g., basketballs, racquetballs, tennis balls (only if used on tile floors), golf balls, etc. Each type of ball will generate a different model. After the activity, discuss with students why the models are different and how the parameters *a* and *b* change with different types of balls.

TECHNOLOGY INTEGRATION

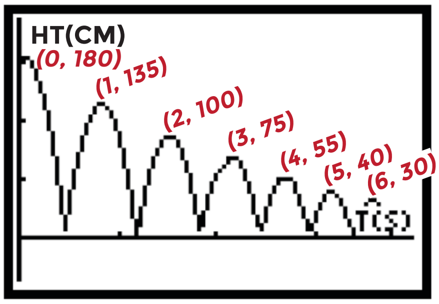
Motion detectors and calculators can be used to capture and graph the height of a ball as it bounces in real time. You can use a motion detector to record the height versus time of a series of bounces, and then use the graph and trace features of the calculator to determine the height of each of the first 5 or 6 bounces and use that data to complete the activity.

- Use the average bounce height as the starting point for the second bounce. Record the bounce height when the ball is released from this drop height for three trials. Calculate the average bounce height.
- Repeat for a total of 6 bounces.
- Record your information in a table like the one shown.

Sample data:

DROP HEIGHT (CM)	BOUNCE HEIGHT 1 (CM)	BOUNCE HEIGHT 2 (CM)	BOUNCE HEIGHT 3 (CM)	AVERAGE BOUNCE HEIGHT (CM)
180	***135***	***130***	***140***	***135***
135	***95***	***105***	***100***	***100***
100	***70***	***70***	***85***	***75***
75	***50***	***55***	***60***	***55***
55	***45***	***35***	***40***	***40***
40	***30***	***25***	***35***	***30***

1. If you were to drop the ball once and let it continue bouncing until it stopped, a height versus time graph of the ball might look like the figure shown. Use an ordered pair (bounce number, height of bounce) to label each point shown on the graph.



2. Calculate the finite differences between the bounce number and the average bounce height. Do the data appear to be linear? How do you know?
See margin.

3. Calculate the ratios between the average bounce heights for successive bounces. Do the data appear to be exponential? How do you know?
See margin.

4. What is the starting point, or *y*-intercept, of the data?
The starting point is 180 centimeters.

2.

+1 +1 +1 +1 +1 +1

BOUNCE NUMBER	0	1	2	3	4	5	6
AVERAGE BOUNCE HEIGHT (CM)	180	135	100	75	55	40	30

-45 -35 -25 -20 -15 -10

The data do not appear to be linear since the finite differences are not constant.

5. Use either the finite differences or common ratios to determine a function that best models the relationship between the bounce number, x, and the average bounce height, $f(x)$. If it is a linear function, use slope-intercept form. If it is an exponential function, use the form $f(x) = ab^x$.
$f(x) = 180(0.75)^x$

6. What do the y-intercept and either rate of change or base from your function rule mean in the context of this situation?
See margin.

7. Use your model to predict the height of the 8th bounce.
Answer using sample data:
$f(8) = 180(0.75)^8 \approx 18$ centimeters

8. What is the diameter of the ball that you used? (You may need to use the formula $C = \pi d$ to calculate the diameter of the ball.)
Answers may vary. A basketball has an approximate diameter of about 24 centimeters.

9. Thinking about the diameter of the ball, what will be the last bounce observed before the ball bounces to a height that is less than the diameter?
See margin.

REFLECT

- **How can you determine an exponential function model for a data set if the common ratios are not exactly the same, but are very close to each other?**
  See margin.
- **Once you have determined your exponential function model, how can you use the model to determine a value of the independent variable that generates a particular value of the dependent variable?**
  See margin.

EXPLAIN

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Exponential function models can be used to represent sets of mathematical and real-world data. If an exponential function is decreasing, then the function is called an **exponential decay** function, since the values of the dependent variable, $f(x)$, decay, or become smaller, as the values of the independent variable, x, increase. The relationship between bounce height and drop height is an exponential decay relationship.

Other exponential functions are increasing and are called **exponential growth** functions. The values of the dependent variable, $f(x)$, grow, or become larger, as the values of the independent variable, x, increase. Population growth is sometimes exponential when the population of a city or county grows at the same percent each year.

6. *The y-intercept, 180 centimeters, represents the height of the ball when it was first dropped.*

The base, 0.75 or 75%, means that the ball will bounce back 75% of the height of the previous bounce.

9. *Answers may vary depending on the ball used and the data collected. For a basketball with the sample data:*

$f(7) = 180(0.75)^7 \approx 24$ centimeters, so the 7th bounce will be the same height as the diameter of the ball, and should be the last "bounce" that is observed.

REFLECT ANSWERS:

Use an average value of the common ratios as the base of an exponential function model.

Make a graph of the exponential function and then locate the ordered pair along the curve that contains the value of the dependent variable, usually y. The x-coordinate of this ordered pair is the value of the independent variable that generates that y-value.

3.

+1 +1 +1 +1 +1 +1

BOUNCE NUMBER	0	1	2	3	4	5	6
AVERAGE BOUNCE HEIGHT (CM)	180	135	100	75	55	40	30

$\frac{135}{180}=0.75$ $\frac{100}{135}\approx 0.74$ $\frac{75}{100}=0.75$ $\frac{55}{75}\approx 0.73$ $\frac{40}{55}\approx 0.73$ $\frac{30}{40}=0.75$

The data appear to be exponential since the successive ratios are very close to a constant of 0.75.

For example, the table below shows the population of Hays County, Texas, for certain years since 1985.

Instead of looking at the finite differences, for an exponential function, take a closer look at the successive ratios.

5-YEAR INTERVAL, x	YEAR	POPULATION, $f(x)$
0	1985	56,225
1	1990	65,767
2	1995	78,956
3	2000	99,070
4	2005	126,470
5	2010	158,289
6	2015	197,298

$\Delta x = 1 - 0 = 1$

$\Delta x = 2 - 1 = 1$

$\Delta x = 3 - 2 = 1$

$\Delta x = 4 - 3 = 1$

$\Delta x = 5 - 4 = 1$

$\Delta x = 6 - 5 = 1$

$\frac{y_n}{y_{n-1}} = \frac{65,767}{56,225} \approx 1.17$

$\frac{y_n}{y_{n-1}} = \frac{78,956}{65,767} \approx 1.20$

$\frac{y_n}{y_{n-1}} = \frac{99,070}{78,956} \approx 1.25$

$\frac{y_n}{y_{n-1}} = \frac{126,470}{99,070} \approx 1.28$

$\frac{y_n}{y_{n-1}} = \frac{158,289}{126,470} \approx 1.25$

$\frac{y_n}{y_{n-1}} = \frac{197,298}{158,289} \approx 1.25$

Data Source: U.S. Census Bureau and Texas Department of State Health Services

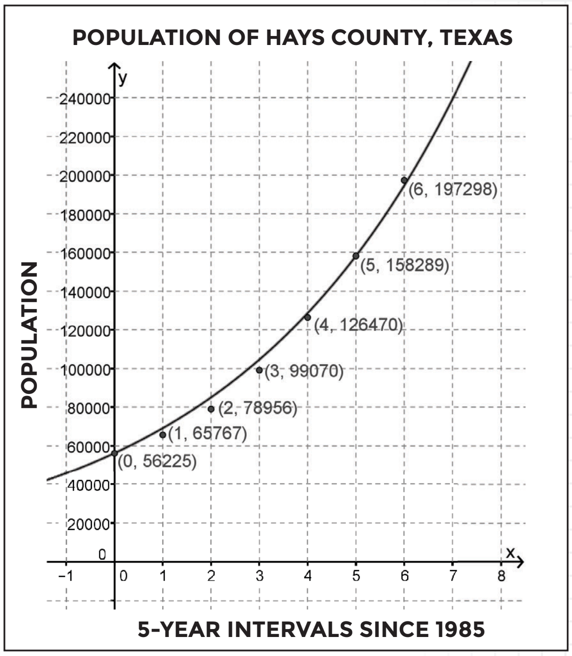
The successive ratios are not equal, but are all close to the same value, 1.25. Calculate the average ratio and use that as the base, b, for the exponential function model.

$$\frac{1.17 + 1.20 + 1.25 + 1.28 + 1.25 + 1.25}{6} \approx 1.23$$

Use the initial population for 5-Year Interval 0, which is 56,225, as the starting point, a, to write the function, $f(x) = 56,225(1.23)^x$. Once you have a function model, you can use that model to make predictions.

$$f(x) = \overset{\text{STARTING POINT}}{\overbrace{56{,}225}}(\underset{\text{COMMON RATIO}}{\underbrace{1.23}})^x$$

Make a scatterplot of the data and graph the function rule over the scatterplot.



Notice that not all of the data points lie on the curve representing the function model. That is because the successive ratios are not exactly equal, but are close to 1.23.

Community leaders need to know how many people will live in the community in order to decide how many fire stations, schools, or restaurants to build. Demographers, or people who study population trends, use population models like this one to make predictions about how many people will live in a community in a particular year. Community and business leaders rely on demographers in order to help them make better decisions for people living in a particular community.

A graph helps you to visualize the data in order to make predictions. For example, interval 7 represents seven 5-year intervals, or 35 years, since 1985. 1985 + 35 = 2020. The population model contains the point (7, 240,000) which means that at interval 7, or the year 2020, the population of Hays County, Texas, could be 240,000.

MODELING WITH EXPONENTIAL FUNCTIONS

Real-world data rarely follows exact patterns, but you can use patterns in data to look for trends. If the data increases or decreases with approximately the same ratio, then an exponential function model may be appropriate for the data set.

Of course, not all exponential relationships involve growth. Automobile depreciation is a loss in the value of an automobile over time. If the value of an automobile loses a percent of its value each year, then the depreciation is an exponential decay.

QUESTIONING STRATEGIES

The scatterplot of data and graphed function rule are shown to the left.

- When looking at the two together, how do you know that the graph is a good representation of the data?
- Which point(s) from the data are included in the graph? (Listen for students to mention the y-intercept.)
- Why is the y-intercept an exact point both from the data and function?

EXAMPLE 1

Approximate radiation levels, in millirads per hour, near the Fukushima nuclear power plant near Naraha, Japan are shown in the table below.

4-DAY INTERVAL, x	DATE	RADIATION LEVEL, $f(x)$
0	MARCH 22, 2011	71.2
1	MARCH 26, 2011	39.8
2	MARCH 30, 2011	22.3
3	APRIL 3, 2011	12.5
4	APRIL 7, 2011	7.1
5	APRIL 11, 2011	3.9
6	APRIL 15, 2011	2.2

Data Source: U.S. Department of Energy

Use the data set to determine if the relationship is linear or exponential.

STEP 1 Determine the finite differences in values of x and values of $f(x)$.

4-DAY INTERVAL, x	DATE	RADIATION LEVEL, $f(x)$
0	MARCH 22, 2011	71.2
1	MARCH 26, 2011	39.8
2	MARCH 30, 2011	22.3
3	APRIL 3, 2011	12.5
4	APRIL 7, 2011	7.1
5	APRIL 11, 2011	3.9
6	APRIL 15, 2011	2.2

$\Delta x = 1 - 0 = 1$ $\quad \Delta f(x) = 39.8 - 71.2 = -31.4$

$\Delta x = 2 - 1 = 1$ $\quad \Delta f(x) = 22.3 - 39.8 = -17.5$

$\Delta x = 3 - 2 = 1$ $\quad \Delta f(x) = 12.5 - 22.3 = -9.8$

$\Delta x = 4 - 3 = 1$ $\quad \Delta f(x) = 7.1 - 12.5 = -5.4$

$\Delta x = 5 - 4 = 1$ $\quad \Delta f(x) = 3.9 - 7.1 = -3.2$

$\Delta x = 6 - 5 = 1$ $\quad \Delta f(x) = 2.2 - 3.9 = -1.7$

ADDITIONAL EXAMPLE

The average salary, in millions of dollars, for professional players on a national sports team is shown in the table below. Use the data set to determine if the relationship is linear or exponential.

Exponential

Encourage students to follow the steps from Example 1 to complete the additional example.

5-YEAR INTERVAL, x	YEAR	SALARY (MILLIONS OF DOLLARS), $f(x)$
0	1995	1.89
1	2000	2.4
2	2005	2.88
3	2010	3.8
4	2015	4.38

STEP 2 Determine the ratios between successive values of $f(x)$.

4-DAY INTERVAL, x	DATE	RADIATION LEVEL, $f(x)$
0	MARCH 22, 2011	71.2
1	MARCH 26, 2011	39.8
2	MARCH 30, 2011	22.3
3	APRIL 3, 2011	12.5
4	APRIL 7, 2011	7.1
5	APRIL 11, 2011	3.9
6	APRIL 15, 2011	2.2

$\Delta x = 1 - 0 = 1$

$\Delta x = 2 - 1 = 1$

$\Delta x = 3 - 2 = 1$

$\Delta x = 4 - 3 = 1$

$\Delta x = 5 - 4 = 1$

$\Delta x = 6 - 5 = 1$

$\frac{y_n}{y_{n-1}} = \frac{39.8}{71.2} \approx 0.559$

$\frac{y_n}{y_{n-1}} = \frac{22.3}{39.8} \approx 0.560$

$\frac{y_n}{y_{n-1}} = \frac{12.5}{22.3} \approx 0.561$

$\frac{y_n}{y_{n-1}} = \frac{7.1}{12.5} \approx 0.568$

$\frac{y_n}{y_{n-1}} = \frac{3.9}{7.1} \approx 0.549$

$\frac{y_n}{y_{n-1}} = \frac{2.2}{3.9} \approx 0.564$

STEP 3 Determine whether the finite differences or the ratios between successive values of $f(x)$ are approximately constant.

- The finite differences range in value from –31.4 to –1.7. This is a wide range, so the finite differences are not even approximately constant.
- The ratios between successive values of $f(x)$ range from 0.549 to 0.568. These values are all close together, so the ratios between successive values of $f(x)$ are approximately constant.

The set of data represents an exponential function, rather than a linear function, because the differences in x are constant and the ratios between successive values of f(x) are approximately constant.

ADDITIONAL EXAMPLE

Students in a Biology class were studying how water evaporates over time. They measured the volume of the water in milliliters each day and recorded the data shown below. Determine if the relationship is linear or exponential.

DAY, x	0	1	2	3	4
WATER LEVEL (ML), $f(x)$	15	13.75	12.5	11.25	10

Exponential

Remind students that the first step is to check the finite differences and then check ratios if the finite differences are not constant.

YOU TRY IT! #1 ANSWER:

The set of data represents a linear function, rather than an exponential function, because the finite differences in x are constant and the finite differences in $f(x)$ are approximately constant.

YOU TRY IT! #1

A major league baseball player's average in successive seasons is recorded in the table.

1-YEAR INTERVAL, x	YEAR	BATTING AVERAGE, $f(x)$
0	2009	0.213
1	2010	0.242
2	2011	0.271
3	2012	0.301
4	2013	0.330
5	2014	0.359

Determine whether the relationship is linear or exponential.
See margin.

EXAMPLE 2

The table below shows the average viewership for the Super Bowl, in numbers of households.

3-YEAR INTERVAL, x	YEAR	VIEWERSHIP, $f(x)$
0	1970	23,050
1	1973	27,670
2	1976	29,440
3	1979	35,090
4	1982	40,020

Data Source: http://www.nielsen.com/us/en.html

Generate an exponential function model for Super Bowl viewership. How many households does your model predict will watch the Super Bowl in the year 2018.

ADDITIONAL EXAMPLE

Have students write the exponential function to match the data that was given in the exponential additional examples on pages 58 and 59.

pg. 58: $f(x) = 1.89(1.24)^x$

pg. 59 $f(x) = 15(0.9)^x$

STEP 1 Determine the finite differences in x-values and the ratios between successive values of $f(x)$.

3-YEAR INTERVAL, x	YEAR	VIEWERSHIP, $f(x)$
0	1970	23,050
1	1973	27,670
2	1976	29,440
3	1979	35,090
4	1982	40,020

$\Delta x = 1 - 0 = 1$

$\Delta x = 2 - 1 = 1$

$\Delta x = 3 - 2 = 1$

$\Delta x = 4 - 3 = 1$

$\frac{y_n}{y_{n-1}} = \frac{27{,}670}{23{,}050} \approx 1.20$

$\frac{y_n}{y_{n-1}} = \frac{29{,}440}{27{,}670} \approx 1.06$

$\frac{y_n}{y_{n-1}} = \frac{35{,}090}{29{,}440} \approx 1.19$

$\frac{y_n}{y_{n-1}} = \frac{40{,}020}{35{,}090} \approx 1.14$

STEP 2 Calculate the average of the ratios and use this value for the base, b, in your exponential function model.

$$\frac{1.20 + 1.06 + 1.19 + 1.14}{4} \approx 1.15$$

STEP 3 Using a, the initial viewership value from the table, and the calculated value for b, write an exponential function model.

$f(x) = 23{,}050(1.15)^x$

STEP 4 Use your exponential function model to predict the 2018 viewership, in number of households.

2018 – 1970 = 48, so the number of 3-year intervals since 1970, $x = 16$.

$f(16) = 23{,}050(1.15)^{16} \approx 215{,}693.2$

According to the exponential function model $f(x) = 23{,}050(1.15)^x$, approximately 215,693 households will view the Super Bowl in the year 2018.

ADDITIONAL EXAMPLE

Alan purchased a new luxury car. He recorded the value of his car each year after his purchase in the table shown. Generate an exponential function model for the value of Alan's car, in dollars.

YEARS SINCE PURCHASE, x	2	3	4	5	6
VALUE (DOLLARS), $f(x)$	41,115	36,181	33,287	30,291	26,353

Using the average ratio of 0.89 to find x = 0 to the nearest whole number, the function would be $f(x) = 51907(0.89)^x$.

Based on your model, how much is Alan's car depreciating in value each year?
Alan's car is decreasing about 11% in value each year.

How much did Alan pay for his car originally?
Alan paid $51,907 for his car.

What will the value of Alan's car be in 10 years?
After 10 years, Alan's car will be worth about $16,186.

YOU TRY IT! #2 ANSWER:

$f(x) = (2.113)^x$; According to the exponential function model, there will be approximately 840 paramecia in the petri dish on the tenth day of the experiment.

YOU TRY IT! #2

A biologist places a single paramecium in a petri dish to observe the rate of population growth of this single-celled organism. The biologist's observations of the number of paramecia in the petri dish over time are recorded in the table below.

1-DAY INTERVAL, x	DAY OF EXPERIMENT	POPULATION, $f(x)$
0	1ST	1
1	2ND	2
2	3RD	5
3	4TH	10
4	5TH	22
5	6TH	44
6	7TH	87

Generate an exponential function model for the paramecium population. How many paramecia does your model predict will be in the petri dish on the tenth day of the experiment?

See margin.

EXAMPLE 3

The population of Throckmorton County, Texas, in each census since 1950 is shown in the table below.

10-YEAR INTERVAL, x	YEAR	POPULATION, $f(x)$
0	1950	3,618
1	1960	2,767
2	1970	2,205
3	1980	2,053
4	1990	1,880
5	2000	1,850
6	2010	1,641

Data Source: U.S. Census Bureau

Use the data set to generate an exponential model. Use your model to predict the population of Throckmorton County, Texas, in the year 2020.

STEP 1 **Determine the finite differences in x-values and the ratios between successive values of $f(x)$.**

10-YEAR INTERVAL, x	YEAR	POPULATION, $f(x)$
0	1950	3,618
1	1960	2,767
2	1970	2,205
3	1980	2,053
4	1990	1,880
5	2000	1,850
6	2010	1,641

$\Delta x = 1 - 0 = 1$

$\Delta x = 2 - 1 = 1$

$\Delta x = 3 - 2 = 1$

$\Delta x = 4 - 3 = 1$

$\Delta x = 5 - 4 = 1$

$\Delta x = 6 - 5 = 1$

$\frac{y_n}{y_{n-1}} = \frac{2,767}{3,618} \approx 0.765$

$\frac{y_n}{y_{n-1}} = \frac{2,205}{2,767} \approx 0.797$

$\frac{y_n}{y_{n-1}} = \frac{2,053}{2,205} \approx 0.931$

$\frac{y_n}{y_{n-1}} = \frac{1,880}{2,053} \approx 0.916$

$\frac{y_n}{y_{n-1}} = \frac{1,850}{1,880} \approx 0.984$

$\frac{y_n}{y_{n-1}} = \frac{1,641}{1,850} \approx 0.887$

STEP 2 **Calculate the average of the ratios and use this value for the base, b, in your exponential function model.**

$$\frac{0.765 + 0.797 + 0.931 + 0.916 + 0.984 + 0.887}{6} \approx 0.88$$

STEP 3 **Using a, the initial population value from the table, and the calculated value for b, write an exponential function model.**

$f(x) = 3618(0.88)^x$

STEP 4 **Use your exponential function model to predict the 2020 population.**

2020 – 1950 = 70, so the number of 10-year intervals since 1950, $x = 7$.

$f(7) = 3618(0.88)^7 \approx 1,478.59$

According to the exponential function model $f(x) = 3618(0.88)^x$, the population of Throckmorton County, Texas, will be approximately 1,479 people in the year 2020.

INSTRUCTIONAL HINTS

Problems like these take mathematical stamina and attention to detail. Have students write the steps they would use to write an exponential function from real world data. Then have students compare this writing to the steps they wrote in the last section for exponential functions with constant ratios. Finally, give students time to briefly explain the steps to one or two other students.

YOU TRY IT! #3

Percentages of adults in the United States who smoke cigarettes on a daily basis are recorded in the table.

2-YEAR INTERVAL, x	YEAR	PERCENTAGE OF AMERICAN ADULTS, $f(x)$
0	1995	19.9
1	1997	19.1
2	1999	18.0
3	2001	17.4
4	2003	16.9
5	2005	15.3
6	2007	14.5

Data Source: Center for Disease Control (CDC)

Use the data set to generate an exponential model. Use your model to predict the percentage of adults in the United States who smoke cigarettes on a daily basis in the year 2015.

See margin.

PRACTICE/HOMEWORK

For questions 1 and 2, determine whether a linear model or an exponential model would be most appropriate for the data. Explain how you made your decision.

1.

x	y
0	45
1	67.5
2	94.5
3	151.2
4	257.04
5	359.856

See margin.

2.

x	y
0	209.5
1	184.6
2	159.6
3	134.4
4	109.6
5	84.5

See margin.

YOU TRY IT! #3 ANSWER:

$f(x) = 19.9(0.949)^x$; According to the exponential function model, approximately 11.8% of American adults will smoke cigarettes on a daily basis in the year 2015.

1. *An exponential model would be best; first difference values range from 22.5 to 102.816, while successive ratio values range from 1.4 to 1.7 (these values are approximately constant).*

2. *A linear model would be best; successive ratio values range from 0.77 to 0.88, while first difference values range from 24.8 to 25.2 (these values are approximately constant).*

For questions 3 - 5, calculate the average ratio between successive y-values.

3.

x	y
0	425.6
1	766.08
2	1225.73
3	2083.74
4	3750.73

1.725

4.

x	0	1	2	3	4	5	6
y	2300.6	1173.31	586.65	287.46	137.98	70.37	36.59

0.5

5.

x	0	1	2	3	4
y	1810.4	2172	2389.2	3105.96	3727.15

1.2

For questions 6 – 8, identify whether the data shows exponential growth or exponential decay. Then, determine an exponential function to model the situation.

SCIENCE

6. The population of gray squirrels in a local park has been recorded every year since 2005.

1-YEAR INTERVAL, x	YEAR	SQUIRREL POPULATION, y
0	2005	62
1	2006	87
2	2007	113
3	2008	170
4	2009	204
5	2010	265

Exponential growth; as time passes, the squirrel population increases.
$y = 62(1.34)^x$, where x represents the number of years since 2005

7. *Exponential growth; as time passes, the value of the painting increases.*

$f(x) = 2200(3.39)^x$, *where x represents the number of 10-year intervals since 1960.*

8. *Exponential decay; as trials numbers increase, the number of pennies decreases.*

$f(x) = 61(0.53)^x$, *where x represents the trial number*

FINANCE

7. Kristal noticed that her favorite painting in a museum has been increasing in value over the years. The changing value of the painting is shown in the table.

10-YEAR INTERVAL, x	YEAR	VALUE OF THE PAINTING, $f(x)$
0	1960	$2200
1	1970	$7500
2	1980	$25,000
3	1990	$86,000
4	2000	$292,000
5	2010	$992,000

See margin.

SCIENCE

8. Mrs. Montgomery's class is doing an experiment with pennies. They empty a cup of pennies onto a table, and remove all the pennies that landed "heads-up." Then, they put the other pennies back in the cup, and repeat the process four more times.

TRIAL NUMBER, x	NUMBER OF PENNIES REMAINING, *fix)*
0	61
1	28
2	16
3	9
4	7
5	2

See margin.

For questions 9 – 12 use the following situation.

FINANCE

Most cars decrease in value over time. The table below shows the value of Carla's car from the time of its purchase.

1-YEAR INTERVAL, x	YEAR	VALUE OF CAR, $f(x)$
0	2007	$29,870
1	2008	$24,180
2	2009	$20,480
3	2010	$17,420
4	2011	$14, 585
5	2012	$12,124

9. Use the data set to generate an exponential model.
$f(x) = 29{,}870\ (0.84)^x$; where x represents the number of years since the car was purchased in 2007

10. What do the y-intercept and base from your function rule mean in context of the situation?
See margin.

11. In what year will the car be worth about $5900?
See margin.

12. Use your model to predict the value of Carla's car in the year 2020.
See margin.

10. *The y-intercept, $29,870, represents the value of the car when Carla purchased it in 2007.*

The base, 0.835, or 84% means that the value of the car is 84% of its value the previous year.

11. $f(x) = 29{,}870(0.835)^x = 5900$, $x \approx 9$ *years;*
2007 + 9 = 2016.

According to the exponential function model, the car will be worth about $5900 in the year 2016.

12. *2020 – 2007 = 13, so the number of years since purchase,* $x = 13$.

$f(13) = 29{,}870(0.835)^{13} \approx$ *$2865.20*

According to the exponential function model, the car will be worth about $2865 in the year 2020.

Using the base of (0.84), the car would be worth $3096 in the year 2020.

15. *$f(x) = 310(1.08)^x = 1000$;*
$x \approx 15$ weeks

According to the exponential function model, Ella will have sold 1,000 ribbons in about 15 weeks.

16. *A year is 52 weeks;*
$f(52) = 310(1.08)^x$;
$x \approx 16{,}959$

According to the exponential function model, Ella will sell close about 16,95 ribbons in a year.

For questions 13 – 16 use the following situation.

FINANCE

Ella sells hair ribbons and decided to start marketing them on the internet hoping to increase her sales. The table shows the total number of ribbons she has sold.

NUMBER OF WEEKS SINCE MARKETING ON THE INTERNET, x	TOTAL NUMBER OF RIBBONS SOLD, $f(x)$
0	310
1	336
2	365
3	388
4	425
5	445
6	496

13. Use the data set to determine an exponential function that models the situation.
$f(x) = 310(1.08)^x$; where x represents the number of weeks since she started marketing her ribbons on the internet.

14. What is the y-intercept of this function, and what does it mean in context of the problem?
The y-intercept, 310, represents the number of ribbons Ella had sold before she started marketing on the internet.

15. Use your function model to determine approximately how many weeks it will take to sell 1,000 ribbons.
See margin.

16. Use your function model to predict how many ribbons Ella will sell in a year.
See margin.

For questions 17 – 20 use the following situation.

SCIENCE

A biologist is recording the population of a certain bacteria in a petri dish. He determines the number of bacteria in the dish every 2 hours, as shown in the table below.

17. Does this data show exponential growth or exponential decay? Explain.
Exponential growth; as time increases, the number of bacteria increases

2-HR INTERVAL, x	NUMBER OF HOURS	NUMBER OF BACTERIA, $f(x)$
0	0	8
1	2	12
2	4	17
3	6	24
4	8	34
5	10	50

18. At some point, it becomes impossible to count all the bacteria, so an equation is necessary. Use the data set to generate an exponential model.
$f(x) = 8(1.44)^x$; where x represents the number of 2-hour time intervals

19. Use your exponential model to determine approximately how many bacteria will be in the petri dish after 1 day.
See margin.

20. Approximately how many days will elapse before there are 451,000 bacteria?
See margin.

For questions 21 – 24 use the following situation.

CRITICAL THINKING

Beth enjoys running for exercise. She has started a training plan that will gradually increase her weekly mileage as she prepares for a half-marathon. The table shows her training plan.

NUMBER OF WEEKS, x	MILES PER WEEK, $f(x)$
0	18
1	20
2	22
3	24
4	26.5
5	29
6	32
7	36

21. Using the data given above, generate an exponential function that models the situation.
$f(x) = 18(1.1)^x$; where x represents the number of weeks on the training plan

22. What is the percent increase of her mileage from week to week on the plan?
b = 1.1 is the average ratio; the 0.10 in this value indicates an increase of 10% each week.

23. According to your model, how much should Beth run in week 10 of her training plan?
See margin.

24. If the training plan limits the mileage to 65 miles per week, when should she reach this goal?
See margin.

19. *1 day is 24 hours, which is 12 2-hr intervals; $x = 12$; $f(12) = 72(0.41)^{12} \approx 636$ bacteria*

According to the exponential function model, there will be about 636 bacteria after 1 day.

20. *$f(x) = 8(1.44)^x = 451{,}000$; $x \approx 30$, and 30 2-hr intervals makes it 60 hours, or 2.5 days*

According to the exponential function model, there will be 451,000 bacteria in about 2.5 days.

23. *$f(10) = 18(1.1)^{10} = 46.7$; $x \approx 46.7$ miles*

According to the exponential function model, she should run about 46.7 miles in week 10.

24. *$f(x) = 18(1.1)^x = 65$; $x \approx 14$ weeks*

According to the exponential function model, she should reach the goal of 65 miles in week 14.