TEKS

AR.2A Determine the patterns that identify the relationship between a function and its common ratio or related finite differences as appropriate, including linear, quadratic, cubic, and exponential functions.

AR.2C Determine the function that models a given table of related values using finite differences and its restricted domain and range.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

ELPS

1A Use prior knowledge and experiences to understand meanings in English.

VOCABULARY

finite differences, slope, y-intercept, linear function, restrictions

MATERIALS

- N/A

ENGAGE ANSWER:

Marcus could place the number of rows in one column and the number of trees in the other column.

1.2 Writing Linear Functions

FOCUSING QUESTION What are the characteristics of a linear function?

LEARNING OUTCOMES

- I can determine patterns that identify a linear function from its related finite differences.
- I can determine the linear function from a table using finite differences, including any restrictions on the domain and range.
- I can analyze patterns to connect the table to a function rule and communicate the linear pattern as a function rule.

ENGAGE

Marcus works in an orchard. Each row in the orchard contains 20 trees. How can Marcus use this information to make a table of values to represent the number of trees in the orchard?

See margin.

EXPLORE

Miranda and her family will spend their summer vacation on the beach. They plan to rent a beach house that has a fixed cleaning fee and a daily rental fee. The table below shows the rental cost for each of a certain number of days.

NUMBER OF DAYS	RENTAL COST
1	\$170
2	\$285
3	\$400
4	\$515
5	\$630
6	\$745
7	\$860

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1. What is the difference between the numbers of days in consecutive rows in the table?
   1 day
2. What is the difference between the rental cost in consecutive rows in the table?
   $115
3. Use the pattern in the table to predict the rental cost for 0 days.
   $55
4. Based on the pattern in the table, what do you think the cleaning fee is? Explain how you know.
   See margin.
5. Based on the pattern in the table, what do you think the daily rental fee is? Explain how you know.
   See margin.
6. Use the pattern in the table to write an equation that shows the relationship between n, the number of days the beach house will be rented and r, the total rental cost.
   r = 55 + 115n

Miranda and her sister have pooled their money for meals. From the initial amount of money they placed in an envelope, they will spend a certain amount each day on food. The table below shows the balance of money remaining in the envelope after a certain number of days.

NUMBER OF DAYS	BALANCE
1	$225
2	$190
3	$155
4	$120
5	$85
6	$50
7	$15

7. What is the difference between the numbers of days in consecutive rows in the table?
   1 day
8. What is the difference between the balances in consecutive rows in the table?
   −$35
9. Use the patterns in the table to predict the balance on day 0.
   $260

4. *The cleaning fee is $55, because that is the amount that you start with on day 0 in the table.*
5. *The daily rental fee is $115, because that is the amount that is added to the total rental cost for each day that Miranda's family rents the beach house.*

QUESTIONING STRATEGIES

As students move through the second scenario, call attention to how they did the first scenario. Ask questions such as:

- How does your answer for this situation compare to the beach house rental?
- Which of the two situations is an increasing function? Decreasing function?

10 *Miranda and her sister initially pooled $260, because that is the amount that you start with on Day 0 in the table.*

11. *Miranda and her sister spent $35 each day on meals, because that is the amount that is subtracted from the balance each day.*

REFLECT ANSWERS:

The differences in values for successive table rows are constant.

The ratio of the differences is the same as the rate of change in the equation.

ELL STRATEGY

Connecting to prior knowledge **(ELPS: c1A)** helps students create understanding of new mathematical topics. Connecting finite differences to independent and dependent variables helps students recall important ideas from Algebra 1 and extend them to new content.

QUESTIONING STRATEGIES

Think back to Miranda and her family.

- Which value(s) represented slope?
- Which value(s) represented the *y*-intercept?

10. Based on the patterns in the table, how much money do you think Miranda and her sister initially pooled? Explain how you know.
See margin.

11. Based on the patterns in the table, how much money did Miranda and her sister spend on meals each day? Explain how you know.
See margin.

12. Use the patterns in the table to write an equation that shows the relationship between n, the number of days of the vacation and b, the balance of pooled money remaining.
b = 260 – 35n

REFLECT

- **What do you notice about the differences in values for successive table rows for both the independent and dependent variable?**
  See margin.
- **What relationship exists between the ratio of the differences in the dependent variable to the differences in the independent variable and the equations that you have written?**
  See margin.

EXPLAIN

The differences in values for successive table rows are called **finite differences**. When you have a table of data, you can use finite differences to determine the type of function the data represents and to write a function representing the relationship between the variables in the table.

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> The slope, which is the rate of change, of a linear function connecting two points, (x_1, y_1) and (x_2, y_2) is found using the slope formula.
>
> $$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's look more closely at a linear function. The table on the next page shows the relationship between x and $f(x)$. In a linear function, $f(x) = mx + b$, m represents the slope or rate of change, and b represents the y-coordinate of the y-intercept, or starting point.

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x	$y = f(x)$
0	b
1	$b + m$
2	$b + 2m$
3	$b + 3m$
4	$b + 4m$
5	$b + 5m$

$\Delta x = 1 - 0 = 1$ | $\Delta y = (b + m) - b = b + m - b = b - b + m = m$

$\Delta x = 2 - 1 = 1$ | $\Delta y = (b + 2m) - (b + m) = b + 2m - b - m = b - b + 2m - m = m$

$\Delta x = 3 - 2 = 1$ | $\Delta y = (b + 3m) - (b + 2m) = b + 3m - b - 2m = b - b + 3m - 2m = m$

$\Delta x = 4 - 3 = 1$ | $\Delta y = (b + 4m) - (b + 3m) = b + 4m - b - 3m = b - b + 4m - 3m = m$

$\Delta x = 5 - 4 = 1$ | $\Delta y = (b + 5m) - (b + 4m) = b + 5m - b - 4m = b - b + 5m - 4m = m$

Notice that in the table, the difference between each pair of x-values, Δx, is 1 and the difference between each pair of y-values is m. The finite differences in y-values, Δy, for a linear function are the same, so we can say that the finite differences are constant.

FINITE DIFFERENCES AND LINEAR FUNCTIONS

In a linear function, the finite differences between successive y-values, Δy, are constant if the differences between successive x-values, Δx, are also constant.

If the finite differences in a table of values are constant, then the values represent a linear function.

You can also use the finite differences to write a linear function describing the relationship between the independent and dependent variables.

EXAMPLE 1

Does the set of data shown below represent a linear function? Justify your answer.

x	y
0	11
1	17
2	23
3	29
4	35

INTEGRATE TECHNOLOGY

Use technology such as a graphing calculator or spreadsheet app on a display screen to show students how, no matter the numbers present in the linear function, the finite differences will always be constant.

INSTRUCTIONAL HINTS

Remind students of prior learning in Algebra I. Students learned multiple ways to test if data represented a linear function.

ADDITIONAL EXAMPLES

Provide students with the lists of ordered pairs of numbers. Have them place the numbers in tables and answer the following question.

Does the data set represent a linear function? Justify your answer.

1. (2, 13), (3, 6), (4, -1), (5, -8), (6, -15)

Yes, the data set represents a linear function because the finite differences in y-values are constant when the finite differences in x-values are also constant.

2. (1, 3), (2, 6), (4, 9), (8, 12), (16, 15)

No, data set does not represent a linear function because while the finite differences in y-values are constant the finite differences in x-values are not constant.

3. (-1, -12), (1, -6.5), (3, -1), (5, 4.5), (7, 10)

Yes, the data set represents a linear function because the finite differences in y-values are constant when the finite differences in x-values are also constant.

Note: Often students forget to check the finite differences between successive x-values. Draw special attention to the second additional example.

YOU TRY IT! #1 ANSWER:

Yes, the set of data represents a linear function, because the finite differences in x-values and the finite differences in y-values are both constant.

STEP 1 Determine the finite differences between successive x-values and successive y-values.

x	y
0	11
1	17
2	23
3	29
4	35

$\Delta x = 1 - 0 = 1$ $\quad \Delta y = 17 - 11 = 6$

$\Delta x = 2 - 1 = 1$ $\quad \Delta y = 23 - 17 = 6$

$\Delta x = 3 - 2 = 1$ $\quad \Delta y = 29 - 23 = 6$

$\Delta x = 4 - 3 = 1$ $\quad \Delta y = 35 - 29 = 6$

STEP 2 Determine whether or not the ratios of the differences are constant.

The differences in x, Δx, are all 1, so they are constant.

The differences in y, Δy, are all 6, so they are constant.

$\frac{\Delta y}{\Delta x} = \frac{6}{1} = 6$ for all pairs of Δx and Δy.

STEP 3 Determine whether or not the set of data represents a linear function.

Yes, the set of data represents a linear function because the finite differences in y-values are constant when the finite differences in x-values are also constant.

YOU TRY IT! #1

Does the set of data shown below represent a linear function? Justify your answer.

x	y
0	6.4
1	7.2
2	8.0
3	8.8
4	9.6

See margin.

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EXAMPLE 2

Does the set of data shown below represent a linear function? Justify your answer.

x	y
0	17
1	15.5
2	14
3	11.5
4	10

STEP 1 **Determine the first differences between successive x-values and successive y-values.**

	x	y	
	0	17	
$\Delta x = 1 - 0 = 1$			$\Delta y = 15.5 - 17 = -1.5$
	1	15.5	
$\Delta x = 2 - 1 = 1$			$\Delta y = 14 - 15.5 = -1.5$
	2	14	
$\Delta x = 3 - 2 = 1$			$\Delta y = 11.5 - 14 = -2.5$
	3	11.5	
$\Delta x = 4 - 3 = 1$			$\Delta y = 10 - 11.5 = -1.5$
	4	10	

STEP 2 **Determine whether or not the differences are constant.**

The differences in x, Δx, are all 1, so they are constant.

The differences in y, Δy, are not all the same, so they are not constant.

STEP 3 **Determine whether or not the set of data represents a linear function.**

No, the set of data does not represent a linear function because the finite differences for the y-values are not constant when the finite differences for the x-values are constant.

QUESTIONING STRATEGIES

Example 2 does not represent a linear function. How does this data look on a scatterplot? How can you tell from looking at the graph, that this is not a linear function?

YOU TRY IT! #2 ANSWER:

No, the set of data does not represent a linear function, because the finite differences in x-values are constant but the finite differences in y-values are not constant.

QUESTIONING STRATEGIES

After reading Example 3's scenario, guide students to plan their steps.

How will the finite differences help you determine the slope?

How can you identify the y-intercept from a table of values?

Do you need to draw a graph to write a function rule?

YOU TRY IT! #2

Does the set of data shown below represent a linear function? Justify your answer.

x	y
0	7.1
1	7.5
2	8.1
3	8.9
4	9.9

See margin.

EXAMPLE 3

For the data set below, determine if the relationship is a linear function. If so, determine a function relating the variables.

x	y
1	7.5
2	10
3	12.5
4	15
5	17.5

STEP 1 Determine the finite differences between successive x-values and successive y-values.

	x	y	
	1	7.5	
$\Delta x = 2 - 1 = 1$			$\Delta y = 10 - 7.5 = 2.5$
	2	10	
$\Delta x = 3 - 2 = 1$			$\Delta y = 12.5 - 10 = 2.5$
	3	12.5	
$\Delta x = 4 - 3 = 1$			$\Delta y = 15 - 12.5 = 2.5$
	4	15	
$\Delta x = 5 - 4 = 1$			$\Delta y = 17.5 - 15 = 2.5$
	5	17.5	

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STEP 2 Determine whether or not the relationship is a linear function.

The differences in x, Δx, are all 1, so they are constant.
The differences in y, Δy, are all 2.5, so they are constant.
Since the finite differences are all constant, the relationship is a linear function.

STEP 3 Determine the slope, or rate of change, of the linear function.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{2.5}{1} = 2.5$$

STEP 4 Determine the y-intercept of the linear function.
Work backwards from $x = 1$ and $y = 7.5$.

x	y
0	b
1	7.5
2	10

$1 - 0 = 1$
$2 - 1 = 1$

$7.5 - b = 2.5$
$10 - 7.5 = 2.5$

$$\begin{aligned} 7.5 - b &= 2.5 \\ 7.5 - 7.5 - b &= 2.5 - 7.5 \\ -b &= -5 \\ b &= 5 \end{aligned}$$

The y-intercept is (0, 5).

STEP 5 Use the slope and the y-coordinate of the y-intercept to write the function in slope-intercept form.

$$y = mx + b$$
$$y = 2.5x + 5$$

ADDITIONAL EXAMPLES

For the following data sets, determine if the relationship is a linear function. If so, determine a function relating the variables.

1.

x	-3	-1	1	3	5
y	0	3	6	9	12

Linear, $y = 1.5x + 4.5$

2.

x	0	1	2	3	4
y	-4	-8	-16	-32	-64

Not Linear

3. (1, 2), (2, 5), (3, 8), (4, 12), (5, 16)

Not Linear

4. (1, -5), (2, -13), (3, -21), (4, -29), (5, -37)

Linear, $y = -8x - 13$

INSTRUCTIONAL HINT

Encourage students to write the finite differences on the table of values for every problem. When $x = 0$, highlight or circle the y-intercept in the table.

YOU TRY IT! #3

For the data set below, determine if the relationship is a linear function. If so, determine a function, in slope-intercept form, relating the variables.

x	y
1	9
2	4
3	-1
4	-6
5	-11

Answer: $y = -5x + 14$

EXAMPLE 4

For the data set below, determine if the relationship is a linear function. If so, write a function, in slope-intercept form, relating the variables.

x	y
2	22
4	21
6	20
8	19
10	18

STEP 1 Determine the finite differences between successive x-values and successive y-values.

Δx	x	y	Δy
	2	22	
$\Delta x = 4 - 2 = 2$			$\Delta y = 21 - 22 = -1$
	4	21	
$\Delta x = 6 - 4 = 2$			$\Delta y = 20 - 21 = -1$
	6	20	
$\Delta x = 8 - 6 = 2$			$\Delta y = 19 - 20 = -1$
	8	19	
$\Delta x = 10 - 8 = 2$			$\Delta y = 18 - 19 = -1$
	10	18	

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STEP 2 Determine whether or not the relationship is a linear function.

The differences in x, Δx, are all 2, so they are constant.
The differences in y, Δy, are all −1, so they are constant.
Since the finite differences are all constant, the relationship is a linear function.

STEP 3 Determine the slope of the linear function.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-1}{2} = -\frac{1}{2}$$

STEP 4 Determine the y-intercept of the linear function.
Work backwards from $x = 2$ and $y = 22$.

	x	y	
	0	b	
$2 - 0 = 2$	2	22	$22 - b = -1$
$4 - 2 = 2$	4	21	$21 - 22 = -1$

$$22 - b = -1$$
$$22 - 22 - b = -1 - 22$$
$$-b = -23$$
$$b = 23$$

The y-intercept is (0, 23).

STEP 5 Use the slope and the y-coordinate of the y-intercept to write the function in slope-intercept form.

$$y = mx + b$$
$$y = -\tfrac{1}{2}x + 23$$

ADDITIONAL EXAMPLES

Some students may need additional practice with identifying slope and y-intercept from an equation and with writing functions given slope and y-intercept.

Identify the slope, m, and y-intercept, b, of the following linear functions.

1. $y = 0.5x + 2$

$m = 0.5,\ b = 2$

2. $y = 3 - 4x$

$m = -4,\ b = 3$

3. $y = -\frac{2}{3}x - 5.5$

$m = -\frac{2}{3},\ b = -5.5$

4. $y = 0.6x$

$m = 0.6,\ b = 0$

5. $y = 7$

$m = 0,\ b = 7$

Write the function given the slope and y-intercept.

1. $m = -0.25,\ b = -3$

$y = -0.25x - 3$

2. slope = $\frac{4}{5}$, contains the point (0, 2)

$y = \frac{4}{5}x + 2$

3. slope = $\frac{1}{2}$, contains the point (0, 0)

$y = \frac{1}{2}x$

4. slope = -4, contains the point (2, -3)

$y = -4x + 5$

5. slope = -0.7, contains the point (10, -10)

$y = -0.7x - 3$

YOU TRY IT! #4

For the data set below, determine if the relationship is a linear function. If so, determine a function relating the variables.

x	y
6	11
9	16
12	21
15	26
18	31

Answer: $y = \frac{5}{3}x + 1$

PRACTICE/HOMEWORK

For questions 1 - 4 determine the equation of the linear function with the given characteristics.

1. slope = 0.4, y-intercept = (0, −3)
$y = 0.4x - 3$

2. slope = $\frac{2}{3}$, y-intercept = $(0, 3\frac{1}{3})$
$y = \frac{2}{3}x + 3\frac{1}{3}$

3. slope = $-\frac{2}{5}$, contains the point (10, 3)
$y = -\frac{2}{5}x + 7$

4. slope = $\frac{1}{4}$, contains the point (−8, 1)
$y = \frac{1}{4}x + 3$

For questions 5 - 16, determine whether or not the relationship shows a linear function. If the data set represents a linear function, write the equation for the function.

5.

x	y
1	1
2	4
3	9
4	16
5	25

not linear

6.

x	y
1	5.5
2	7.5
3	9.5
4	11.5
5	13.5

linear; $y = 2x + 3.5$

7.

x	y
1	8
2	11
3	14
4	17
5	20

linear; $y = 3x + 5$

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8.

x	y
1	24
2	20
3	16
4	12
5	8

linear; $y = -4x + 28$

9.

x	y
0	1.7
1	1.1
2	0.5
3	−0.1
4	−0.7

linear; $y = -0.6x + 1.7$

10.

x	y
0	2
2	4
4	8
6	16
8	32

not linear

11.

x	y
2	4
4	5
6	7
8	10
10	14

not linear

12.

x	y
2	8
4	9
6	10
8	11
10	12

linear; $y = \frac{1}{2}x + 7$

13.

x	y
3	2
5	10
7	18
9	26
11	34

linear; $y = 4x - 10$

14.

x	y
1	10
2	8
3	6
4	4
5	2

linear; $y = -2x + 12$

15.

x	y
1	16
2	15
3	13
4	10
5	6

not linear

16.

x	y
1	120
2	60
3	40
4	30
5	24

not linear

For questions 17 - 20, use the information in the problem to create a table of data. Then, use the table to determine if the situation is linear or not. If the situation is linear, then use the table to determine a linear function.

SCIENCE

17. The elevation of Lake Sam Rayburn is 164 feet above mean sea level. During the summer, if it does not rain, the elevation of the lake decreases by 0.5 feet each week.

x	y
0	*164*
1	*163.5*
2	*163*
3	*162.5*
4	*162*

$\Delta x = 1$ and $\Delta y = -0.5$, so the situation is linear.

$y = -0.5x + 164$

23

18. A swimming pool has a capacity of 10,000 gallons of water. The swimming pool was about 20% full when a water hose was turned on to fill the pool at a rate of 75 gallons every 5 minutes.

x	y
0	2,000
5	2,075
10	2,150
15	2,225
20	2,300

Δx = 5 and Δy = 75, so the situation is linear.

y = 15x + 2,000

19. According to a recent county health department survey, there were 750 mosquitos per acre in a county park. After a recent rainstorm, the number of mosquitos doubled every 2 days.

x	y
0	750
2	1,500
4	3,000
6	6,000
8	12,000

Δx = 2 but Δy = is not constant, so the situation is not linear.

FINANCE

20. Marla has $85 in her savings. She earns $6.50 per hour after taxes and payroll deductions and plans to save half of what she earns each hour.

x	y
0	85.00
1	88.25
2	91.50
3	94.75
4	98.00

Δx = 1 and Δy = 3.25, so the situation is linear.

y = 3.25x + 85

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