Writing Linear Equations

Linear Function

- A linear function can be described by a linear equation.
- You can use function notation to show that the output value of the function f, written $f(x)$, corresponds to the input value x.

Linear Function

- Linear functions are functions with a constant rate of change and can be written in the form $f(x) = mx + b$, where x is the independent variable and m and b are constants.
- The graph of a linear function is a straight line made up of the set of all points that satisfy $y = f(x)$.

Slope-Intercept Form

- An equation written in the form y = mx + b

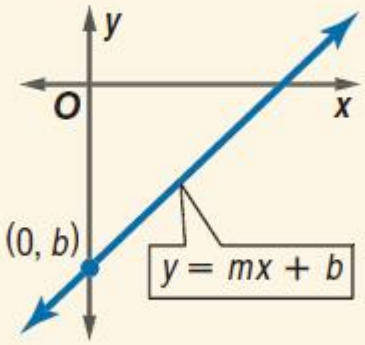
KEY CONCEPT — **Slope-Intercept Form of a Linear Equation**

Words The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

Symbols $y = mx + b$

slope — m; y-intercept — b

Model



Examples

- Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{2}$ and passes through (-4, 1)

Examples

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- $y = mx + b$
- $1 = -\frac{3}{2}(-4) + b$
- $1 = 6 + b$
- $-5 = b$
- $y = -\frac{3}{2}x - 5$

Examples

- Write an equation in slope-intercept form for the line that has a slope of $\frac{4}{3}$ and passes through (3, 2)

Examples

- Write an equation in slope-intercept form for the line that has a slope of $\frac{4}{3}$ and passes through (3, 2)

- $y = mx + b$
- $2 = \frac{4}{3}(3) + b$
- $2 = 4 + b$
- $-2 = b$
- $y = \frac{4}{3}x - 2$

Examples

- Write an equation in slope-intercept form for the line that has a slope of -4 and passes through (-2, -2)

Examples

- Write an equation in slope-intercept form for the line that has a slope of -4 and passes through (-2, -2)

- $y = mx + b$
- $-2 = -4(-2) + b$
- $-2 = 8 + b$
- $-10 = b$
- $y = -4x - 10$

Point-Slope Form

- An equation written in the form $y - y_1 = m(x - x_1)$

KEY CONCEPT — **Point-Slope Form of a Linear Equation**

Words The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

Symbols

$$y - y_1 = m(x - x_1)$$

slope

coordinates of point on line

Examples

- Write an equation in slope-intercept form for the line that passes through (2, 3) and (-4, -5)

Examples

- Write an equation in slope-intercept form for the line that passes through (2, 3) and (-4, -5)

- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- $m = \frac{-5-3}{-4-2} = \frac{-8}{-6} = \frac{4}{3}$
- $y - y_1 = m(x - x_1)$
- $y - 3 = \frac{4}{3}(x - 2)$
- $y - 3 = \frac{4}{3}x - \frac{8}{3}$

$y = \frac{4}{3}x - \frac{8}{3} + 3$

$y = \frac{4}{3}x - \frac{8}{3} + \frac{9}{3}$

$y = \frac{4}{3}x + \frac{1}{3}$

Examples

- Write an equation in slope-intercept form for the line that passes through (6, 1) and (8, -4)

Examples

- Write an equation in slope-intercept form for the line that passes through (6, 1) and (8, -4)

- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- $m = \frac{-4 - 1}{8 - 6} = \frac{-5}{2}$
- $y - y_1 = m(x - x_1)$
- $y - 1 = \frac{-5}{2}(x - 6)$
- $y - 1 = \frac{-5}{2}x + \frac{30}{2}$

$y = \frac{-5}{2}x + 15 + 1$

$y = \frac{-5}{2}x + 16$

Parallel and Perpendicular Lines

- We can use the slope-intercept and point-slope forms to write equations of lines that are parallel or perpendicular to other lines.

Parallel Lines

- Parallel lines are lines that lie in the same plane and do not intersect.
- Parallel lines have the same slope.

Examples

- Write an equation for the line that passes through (4, 6) and is parallel to the graph of $y = \frac{2}{3}x + 5$.

Examples

- Write an equation for the line that passes through (4, 6) and is parallel to the graph of $y = \frac{2}{3}x + 5$.

- $m = \frac{2}{3}$, p(4, 6)
- $y - y_1 = m(x - x_1)$
- $y - 6 = \frac{2}{3}(x - 4)$
- $y - 6 = \frac{2}{3}x - \frac{8}{3}$

$y = \frac{2}{3}x - \frac{8}{3} + 6$

$y = \frac{2}{3}x - \frac{8}{3} + \frac{18}{3}$

$y = \frac{2}{3}x + \frac{10}{3}$

Perpendicular Lines

- Perpendicular lines are lines that lie in the same plane and intersect at right angles.
- Two lines are perpendicular if the product of their slopes is -1.
- Two lines are perpendicular if the slopes are negative reciprocals.

Examples

- Write the equation for the line that passes through (-3, -2) and is perpendicular to the graph of $x + 4y = 12$.

- Find the slope: $-\frac{1}{4}x$
- Find the negative reciprocal: 4
- Use point-slope form: $y - y_1 = m(x - x_1)$
- $y + 2 = 4(x + 3)$
- $y = 4x + 12 - 2$
- $y = 4x + 10$