TRANSFORMING AND ANALYZING EXPONENTIAL FUNCTIONS

EXPONENTIAL FUNCTIONS

- An exponential function is a function that uses a constant multiplier, or base, to show either growth or decay.

- For an exponential function, the general form is $f(x) = a(b)^{kx-c} + d$, where a, k, c, and d are real numbers.

- The exponential parent function is $f(x) = b^x$.

- The full family of exponential functions is generated by applying transformations to the exponential parent function

- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

CHANGES IN *A*

- The parameter a influences the vertical stretch or compression of the graph of the parabola.

- If $|a| > 1$, then the y-values are multiplied by a factor of a to vertically stretch the graph

- If $0 < |a| < 1$, then the y-values are multiplied by a factor of a to vertically compress the graph

- If $a < 0$, then the graph will be reflected across the x-axis

CHANGES IN K

- The parameter k influences the horizontal stretch or compression of the graph.

- If $|k| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|k|}$ to horizontally compress the graph

- If $0 < |k| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|k|}$ to horizontally stretch the graph

- If $k < 0$, then the graph will be reflected across the y-axis

CHANGES IN C

- The parameter c, like *k*, influences the horizontal translation of the graph.

- Note that in the general form, the sign in front of the c is negative. This means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation.

- If *c* > 0, then the graph will translate $|\frac{c}{k}|$ to the right.

- If *c* < 0, then the graph will translate $|\frac{c}{k}|$ to the left.

CHANGES IN *D*

- The parameter *d* influences the vertical translation of the graph.

- If *d* > 0, then the graph of the parabola will translate |*d*| units up.

- If *d* < 0, then the graph of the parabola will translate |*d*| units down.

ASYMPTOTES

- Each exponential function has one horizontal asymptote

- The horizontal asymptote is governed by vertical parameters changes. A vertical translation moves the asymptote d units and a vertical dilation does not move the asymptote.

  - horizontal asymptote: y = d

DOMAIN AND RANGE

- A exponential function does not have any domain restrictions. Therefore, the domain will always be *all real numbers*, or $\{x \mid x \in \mathbb{R}\}$

- The range is restricted by the horizontal asymptote, $y = d$. If $a > 0$, then the range is $y > d$. If $a < 0$, then the range is $y < d$.
  - $a > 0$, $\{y \mid y > d\}$
  - $a < 0$, $\{y \mid y < d\}$

X- AND Y-INTERCEPTS

- An exponential function has at most one x-intercepts. Use the graph and the calculator to determine the value of the x-intercept

- An exponential function has at most one y-intercepts. If it exists, the y-intercept is located at:

  - $(0, \frac{a}{b^c} + d)$

EXAMPLES

- What transformations of the exponential parent function, $f(x) = 10^x$, will result in the graph of the exponential function $g(x) = -3(10)^{2x-1} + 5$?

EXAMPLES

- Step 1: Determine the values of the parameters a, k, c, and d of g(x) and the value of b, the base of g(x).

  - a = -3, k = 2, c = 1, and d = 5

EXAMPLES

- Use the values of the parameters to describe the transformations of the exponential parent function f(x) that are necessary to produce g(x).

- a = -3; vertical stretch by a factor of 3, reflected over the x-axis

- k = 2; horizontal compression by a factor of $\frac{1}{2}$

- c = 1; horizontal shift $\frac{1}{2}$ unit to the right

- d = 5; vertical shift 5 units up

EXAMPLES

- Identify the key attributes of $y = -2^{1.5x-3} + 1$, including domain and range, asymptote, x-intercept, and y-intercept. Write the domain and range in set builder notation.

EXAMPLES

- Step 1: Determine the domain, range and asymptote of $y = -2^{1.5x-3} + 1$.

  - The domain is all real numbers; $\{x \mid x \in \mathbb{R}\}$

  - The range is affected by a and d; a is negative, d = 1

  - The range is numbers < 1; $\{y \mid y \leq 1\}$

  - Asymptote: y = 1

EXAMPLES

- Step 2: Determine if the function has an x-intercept

  - The function has an x-intercept at (2, 0)

- Step 3: Determine if the function has a y-intercept

  - The function has a y-intercept at $(0, \frac{7}{8})$