



# Transforming and Analyzing Absolute Value Functions

# Absolute Value Functions

- For an Absolute Value function, the general form is  $f(x) = a|bx - c| + d$ , where  $a$ ,  $b$ ,  $c$ , and  $d$  are real numbers.
- The Absolute Value parent function is  $f(x) = |x|$
- The full family of Absolute Value functions is generated by applying transformations to the Absolute Value parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

# Changes in $a$

- The parameter  $a$  influences the vertical stretch or compression of the graph of the parabola.
- If  $|a| > 1$ , then the  $y$ -values are multiplied by a factor of  $a$  to vertically stretch the graph
- If  $0 < |a| < 1$ , then the  $y$ -values are multiplied by a factor of  $a$  to vertically compress the graph
- If  $a < 0$ , then the graph will be reflected across the  $x$ -axis

## Changes in $b$

- The parameter  $b$  influences the horizontal stretch or compression of the graph of the parabola.
- If  $|b| > 1$ , then the x-values are multiplied by a factor of  $\frac{1}{|b|}$  to horizontally compress the graph
- If  $0 < |b| < 1$ , then the x-values are multiplied by a factor of  $\frac{1}{|b|}$  to horizontally stretch the graph
- If  $b < 0$ , then the graph will be reflected across the y-axis

## Changes in $c$

- The parameter  $c$ , like  $b$ , influences the horizontal translation of the graph of the parabola.
- Note that in the general form, the sign in front of the  $c$  is negative. This means that when reading the value of  $c$  from the equation, you should read the opposite sign from what is given in the equation.
- If  $c > 0$ , then the graph will translate  $|\frac{c}{b}|$  to the right.
- If  $c < 0$ , then the graph will translate  $|\frac{c}{b}|$  to the left.

## Changes in $d$

- The parameter  $d$  influences the vertical translation of the graph of the parabola.
- If  $d > 0$ , then the graph of the parabola will translate  $|d|$  units up.
- If  $d < 0$ , then the graph of the parabola will translate  $|d|$  units down.

# Vertex

- The vertex of a parabola is a maximum or minimum value.
- If the parabola opens up, then the vertex is a minimum value. If the parabola opens down, then the vertex is a maximum value.
- The location of the vertex is
  - $(\frac{c}{b}, d)$

# Domain and Range

- A Absolute Value function involves squaring a number. Since every real number can be squared, there are no domain restrictions. Therefore, the domain will always be *all real numbers*, or  $\{x \mid x \in \mathbb{R}\}$
- The range does have restrictions. The range is affected by parameters  $a$  and  $d$ . If  $a > 0$ , then  $d$  sets the  $y$ -coordinate of the vertex at a minimum value. The range becomes  $y \geq d$  or  $\{f(x) \mid f(x) \geq d\}$
- If  $a < 0$ , then  $d$  sets the  $y$ -coordinate of the vertex at a maximum value. The range becomes  $y \leq d$  or  $\{f(x) \mid f(x) \leq d\}$



# X- and Y- intercepts

- A Absolute Value function has as many as two x-intercepts, also called *zeroes*. The x-intercepts are located at:

- $(\frac{c \pm \frac{d}{a}}{b}, 0)$

- If the is in the general form,  $y = a|bx - c| + d$  then we find the y-intercept by substituting  $x = 0$ :
  - the y-intercept becomes  $(0, a|c| + d)$

# Examples

- What transformations of the absolute value parent function,  $f(x) = |x|$ , will result in the graph of the absolute value function  $g(x) = -\frac{1}{2}|2x + 1| - 3$ ?

# Examples

- Step 1: Rewrite the equation of  $g(x)$  in general form to determine the values of the parameters  $a$ ,  $b$ ,  $c$ , and  $d$ .
  - $g(x) = a(bx - c)^2 + d$
  - $g(x) = \frac{1}{3}(x - 1)^2 - 4$
  - $g(x) = \frac{1}{3}(x - 1)^2 + (-4)$
- So,  $a = \frac{1}{3}$ ,  $b = 1$ ,  $c = 1$ , and  $d = -4$

# Examples

- Step 2: Use the values of the parameters to describe the transformations of the Absolute Value parent function  $f(x)$  that are necessary to produce  $g(x)$ .
- $a = \frac{1}{3}$ ; so  $|a| < 1$ , then the  $y$ -values are **multiplied by a factor of  $\frac{1}{3}$**  to vertically compress the graph
- $b = 1$ ; there is no affect to the graph
- $c = 1$ , so  $c > 0$ , then the graph will **translate  $|1| = 1$  to the right**
- $d = -4$ , so  $d < 0$ , then the graph of the parabola will **translate  $|4|$  units down**

# Examples

- Identify the key attributes of  $f(x) = \frac{5}{4}|x + 2| - 1$ , including domain, range, vertex, x- and y-intercepts. Write the domain and range as intervals and in set builder notation. Determine whether the vertex is a maximum or a minimum value of the function.

# Examples

- Step 1: Determine the domain and range of  $f(x) = \frac{5}{4}|x + 2| - 1$ . The domain is always *all real numbers*
  - $(-\infty, \infty)$
  - $\{x | x \in \mathbb{R}\}$
- Since  $a > 0$ , the graph will open up. So the range will be numbers  $f(x) > -1$ 
  - $(-1, \infty)$
  - $\{f(x) | f(x) \geq -1\}$

# Examples

- Step 2: Determine the vertex of the parabola.
  - The vertex is  $(\frac{c}{b}, d)$
  - $(\frac{-2}{1}, -1) = (-2, -1)$
- Since  $a > 0$ , this value is a minimum

# Examples

- Step 3: Determine the x-intercepts.
  - The x-intercepts are located at  $(\frac{c \pm \frac{d}{a}}{b}, 0)$ 
    - $(\frac{-2 \pm \frac{-1}{1.25}}{1}, 0)$
    - $(\frac{-2 \pm \frac{4}{5}}{1}, 0)$
    - $(-2 \pm \frac{4}{5}, 0)$
    - $(-2 + \frac{4}{5}, 0); (-2 - \frac{4}{5}, 0)$
    - $(-2 + \frac{4}{5}, 0); (-2 - \frac{4}{5}, 0)$
    - $(-1.2, 0); (-2.8, 0)$



# Examples

- Step 4: Determine the y-intercepts
  - The y-intercept occurs where  $x = 0$ 
    - $f(x) = \frac{5}{4}|x + 2| - 1$
    - $f(x) = \frac{5}{4}|0 + 2| - 1$
    - $f(x) = \frac{5}{4}|2| - 1$
    - $f(x) = \frac{10}{4} - 1$
    - $f(x) = \frac{6}{4} = 1.5$ 
      - $(0, 1.5)$