Transforming and Analyzing Cubic Functions

Cubic Functions

- For a cubic function, the general form is $f(x) = a(bx - c)^3 + d$, where a, b, c, and d are real numbers.
- The cubic parent function is $f(x) = x^3$
- The full family of cubic functions is generated by applying transformations to the cubic parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

Changes in a

- The parameter a influences the vertical stretch or compression of the graph of the parabola.
- If $|a| > 1$, then the y-values are multiplied by a factor of a to vertically stretch the graph
- If $0 < |a| < 1$, then the y-values are multiplied by a factor of a to vertically compress the graph

Changes in *b*

- The parameter b influences the horizontal stretch or compression of the graph of the parabola.

- If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph

- If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph

Changes in *b*

- If b < 0, the all of the x-values will change signs and the parabola will be reflected across the y-axis.
- Since the parabola has a vertical axis of symmetry, the horizontal reflection is not noticeable in the graph.

Changes in c

- The parameter c, like b, influences the horizontal translation of the graph of the parabola.
- Note that in the general form, the sign in front of the c is negative. This means that when reading the value of c from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $|\frac{c}{b}|$ to the right.
- If $c < 0$, then the graph will translate $|\frac{c}{b}|$ to the left.

Changes in d

- The parameter d influences the vertical translation of the graph of the parabola.
- If $d > 0$, then the graph of the parabola will translate $|d|$ units up.
- If $d < 0$, then the graph of the parabola will translate $|d|$ units down.

Domain and Range

- A cubic function involves cubing a number. Since every real number can be cubed, there are no domain restrictions. Therefore, the domain will always be *all real numbers*, or
  - $\{x \mid x \in \mathbb{R}\}$
- The range of a cubic function comes from a set of cubed numbers. When you multiply three positive numbers, the product is positive. When you multiply three negative numbers, the product is negative. Therefore, the range will always be *all real numbers*, or
  - $\{y \mid y \in \mathbb{R}\}$

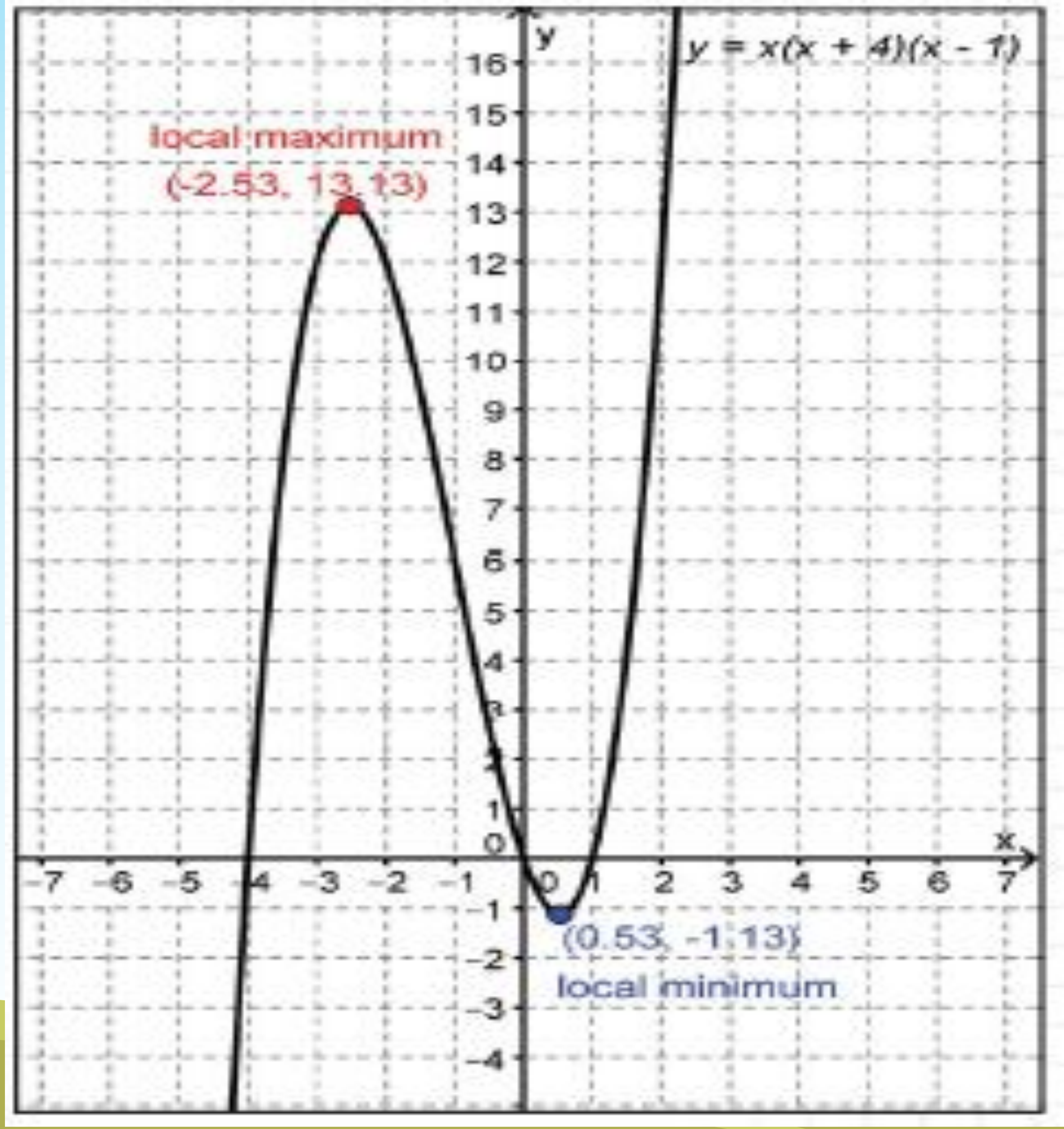
x- and y-intercept

- A cubic equation can have as many as three x-intercepts
- Use a graphing calculator to find your x-intercepts

- If the equation of the parabola is in the general form, $y = a(bx - c)^3 + d$ then we find the y-intercept by substituting $x = 0$:
    - the y-intercept becomes $(0, -ac^3 + d)$

Maximum and Minimum values

Because the range of any cubic function is *all real numbers*, there is not an absolute maximum or minimum value for a cubic function. However, for some cubic functions, there are local maximum or minimum values.



Examples

- What transformations of the cubic parent function, $f(x) = x^3$, will result in the graph of the cubic function $g(x) = \frac{1}{3}(-6x - 2)^3 + 1$?

Examples

- Step 1: Rewrite the equation of g(x) in general form to determine the values of the parameters a, b, c, and d.
  - $g(x) = a(bx - c)^3 + d$
  - $g(x) = \frac{1}{3}(-6x - 2)^3 + 1$
  - So, $a = \frac{1}{3}$, $b = -6$, $c = 2$, and $d = 1$

Examples

- Step 2: Use the values of the parameters to describe the transformations of the Cubic parent function f(x) that are necessary to produce g(x).

- $a = \frac{1}{3}$; so $|a| > 1$, then the y-values are multiplied by a factor of $\frac{1}{3}$ to vertically compress the graph

- $b = -6$; so $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{6}$ to horizontally compress the graph. Also, since $b < 0$, the graph will be reflected over the y-axis

- $c = 2$, so $c > 0$, then the graph will translate $\left|\frac{2}{6}\right| = \frac{1}{3}$ to the right

- $d = 1$, so $d > 0$, then the graph of the parabola will translate 1 units up

Examples

- Identify the key attributes of $f(x) = \frac{3}{4}(0.2x + 5)^2 - 1$, including domain, range, vertex, x- and y-intercepts. Write the domain and range as intervals and as inequalities.

Examples

- Step 1: Determine the domain and range of $f(x) = \frac{3}{4}(0.2x + 5)^2 - 1$
- The domain is always *all real numbers*
    - $(-\infty, \infty)$
    - $-\infty < x < \infty$
- The range is always *all real numbers*
    - $(-\infty, \infty)$
    - $-\infty < y < \infty$

Examples

- Step 2: Determine the x-intercepts.
  - Use your calculator to find your x-intercepts
    - (-19.5, 0)

- Step 3: Determine the y-intercepts.
  - $(0, -ac^3 + d)$
  - $(0, -\frac{3}{4} * (-5)^3 - 1)$
  - (0, 92.75)

Examples

- Identify and compare the x-intercepts of $f(x) = x - 1$, $g(x) = (x + 3)^2$, and $h(x) = (x - 1)(x + 3)^2$.

Examples

- Step 1: Determine the x-intercepts of f(x).
  - Since f(x) is linear, it has one x-intercept at $(\frac{ac-d}{ab}, 0)$
    - $(\frac{1*0-(-1)}{1*1}, 0) = (1, 0)$

Examples

- Step 2: Determine the x-intercepts of g(x).

- Since f(x) is a quadratic function, it will have only one x-intercept at $(\frac{c \pm \sqrt{(\frac{-d}{a})}}{b}, 0)$

  - $(\frac{-3 \pm \sqrt{(\frac{-0}{1})}}{1}, 0)$
  - $(\frac{-3 \pm 0}{1}, 0)$
  - $(-3, 0)$

Examples

- Step 3: Determine the x-intercepts of h(x).
- Using a graphing calculator, the x-intercepts are found at (-3, 0) and (1, 0)

Examples

- Step 4: Compare the x-intercepts of f(x), g(x) and h(x).
  - f(x) has only one x-intercept
  - g(x) has only one x-intercept
  - One of the intercepts of h(x) is the same as f(x), and the other is the same as g(x).