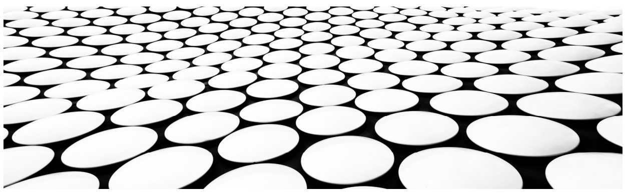
TRANSFORMING AND ANALYZING QUADRATIC FUNCTIONS



QUADRATIC FUNCTIONS

- For a quadratic function, the general form is $f(x) = a(bx - c)^2 + d$, where a, b, c, and d are real numbers.
- The quadratic parent function is $f(x) = x^2$
- The full family of quadratic functions is generated by applying transformations to the quadratic parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

CHANGES IN *A*

- The parameter *a* influences the vertical stretch or compression of the graph of the parabola.
- If $|a| > 1$, then the y-values are multiplied by a factor of *a* to vertically stretch the graph
- If $0 < |a| < 1$, then the y-values are multiplied by a factor of *a* to vertically compress the graph

CHANGES IN *B*

- The parameter *b* influences the horizontal stretch or compression of the graph of the parabola.
- If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph
- If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph

CHANGES IN *B*

- If b < 0, the all of the x-values will change signs and the parabola will be reflected across the y-axis.
- Since the parabola has a vertical axis of symmetry, the horizontal reflection is not noticeable in the graph.

CHANGES IN *C*

- The parameter *c*, like *b*, influences the horizontal translation of the graph of the parabola.
- Note that in the general form, the sign in front of the *c* is negative. This means that when reading the value of *c* from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $|\frac{c}{b}|$ to the right.
- If $c < 0$, then the graph will translate $|\frac{c}{b}|$ to the left.

CHANGES IN *D*

- The parameter *d* influences the vertical translation of the graph of the parabola.
- If $d > 0$, then the graph of the parabola will translate $|d|$ units up.
- If $d < 0$, then the graph of the parabola will translate $|d|$ units down.

VERTEX

- The vertex of a parabola is a maximum or minimum value.
- If the parabola opens up, then the vertex is a minimum value. If the parabola opens down, then the vertex is a maximum value.
- The location of the vertex is
  - $(\frac{c}{b}, d)$

DOMAIN AND RANGE

- A quadratic function involves squaring a number. Since every real number can be squared, there are no domain restrictions. Therefore, the domain will always be *all real numbers*, or $\{x \mid x \in \mathbb{R}\}$
- The range does have restrictions. The range is affected by parameters a and d. If $a > 0$, then d sets the y-coordinate of the vertex at a minimum value. The range becomes $y \geq d$ or $\{f(x) \mid f(x) \geq d\}$
- If $a < 0$, then d sets the y-coordinate of the vertex at a maximum value. The range becomes $y \leq d$ or $\{f(x) \mid f(x) \leq d\}$

X- AND Y-INTERCEPTS

- A quadratic function has as many as two x-intercepts, also called *zeroes*. The x-intercepts are located at:
  - $(\frac{c \pm \sqrt{(\frac{-d}{a})}}{b}, 0)$
- If the equation of the parabola is in the general form, $y = a(bx - c) + d$ then we find the y-intercept by substituting $x = 0$:
  - the y-intercept becomes $(0, ac^2 + d)$

EXAMPLES

- What transformations of the quadratic parent function, $f(x) = x^2$, will result in the graph of the quadratic function $g(x) = \frac{1}{3}(x - 1)^2 - 4$?

EXAMPLES

- Step 1: Rewrite the equation of g(x) in general form to determine the values of the parameters a, b, c, and d.
  - $g(x) = a(bx - c)^2 + d$
  - $g(x) = \frac{1}{3}(x - 1)^2 - 4$
  - $g(x) = \frac{1}{3}(x - 1)^2 + (-4)$
- So, $a = \frac{1}{3}$, $b = 1$, $c = 1$, and $d = -4$

EXAMPLES

- Step 2: Use the values of the parameters to describe the transformations of the quadratic parent function f(x) that are necessary to produce g(x).
- a = $\frac{1}{3}$; so |a| > 1, then the y-values are multiplied by a factor of $\frac{1}{3}$ to vertically compress the graph
- b = 1; there is no affect to the graph
- c = 1, so *c* < 0, then the graph will translate |1| = 1 to the right
- d = -4, so *d* < 0, then the graph of the parabola will translate |4| units down

EXAMPLES

- Identify the key attributes of f(x) = $\frac{2}{3}(x + 4)^2 + 1$, including domain, range, vertex, x- and y-intercepts. Write the domain and range as intervals and in set builder notation. Determine whether the vertex is a maximum or a minimum value of the function.

EXAMPLES

- Step 1: Determine the domain and range of f(x) = $\frac{2}{3}(x + 4)^2 + 1$
  - The domain is always *all real numbers*
    - $(-\infty, \infty)$
    - $\{x | x \in \mathbb{R}\}$
  - Since a > 0, the graph will open up. So the range will be numbers f(x) > 1
    - $(1, \infty)$
    - $\{f(x) | f(x) \geq 1)$

EXAMPLES

- Step 2: Determine the vertex of the parabola.
  - The vertex is $(\frac{c}{b}, d)$
  - $(\frac{-4}{1}, 1)$ = (-4, 1)
  - Since a > 0, this value is a minimum

EXAMPLES

- Step 3: Determine the x-intercepts.
- Since the minimum value is 1, this function cannot have x-intercepts because the graph will never touch the x-axis

EXAMPLES

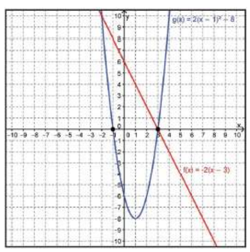
- Step 4: Determine the y-intercepts
  - The y-intercept occurs where x = 0
  - $(0, ac^2 + d)$
  - $(0, \frac{2}{3} * (-4)^2 + 1)$
  - $(0, \frac{2}{3} * 16 + 1)$
  - $(0, \frac{32}{3} + \frac{3}{3})$
  - $(0, \frac{35}{3})$ or $(0, 11\frac{2}{3})$

EXAMPLES

- Identify and compare the x-intercept(s) of f(x) = -2(x – 3) and the x-intercept(s) of g(x) = 2(x – 1)^2 – 8.

EXAMPLES

- Step 1: Graph both f(x) and g(x) to visually estimate the x-intercepts.



EXAMPLES

- Step 2: Determine the x-intercepts of f(x).
  - Since f(x) is a linear function, it will have only one x-intercept at ($\frac{ac-d}{ab}$, 0)
    - ($\frac{-2*3-0}{-2*1}$, 0) = ($\frac{-6}{-2}$, 0) = (3, 0)

EXAMPLES

- Step 3: Determine the x-intercepts of g(x). Since g(x) is quadratic, the x-intercepts are at $(\frac{c \pm \sqrt{(\frac{-d}{a})}}{b}, 0)$.
  - $(\frac{c \pm \sqrt{(\frac{-d}{a})}}{b}, 0) = (\frac{1 \pm \sqrt{(\frac{-(-8)}{2})}}{1}, 0) = (\frac{1 \pm \sqrt{(\frac{8}{2})}}{1}, 0) = (\frac{1 \pm \sqrt{4}}{1}, 0) = (\frac{1 \pm 2}{1}, 0)$
    - $= (\frac{1+2}{1}, 0) = (3, 0)$
    - $= (\frac{1-2}{1}, 0) = (-1, 0)$

EXAMPLES

- Step 4: Compare the x-intercepts of f(x) and g(x).
  - Since f(x) is a linear function, it has only one x-intercept, (3, 0).
  - G(x) is quadratic and has two x-intercepts, (-1, 0) and (3, 0).
    - One of the intercepts of g(x) is the same as f(x).