Transforming and Analyzing Linear Functions

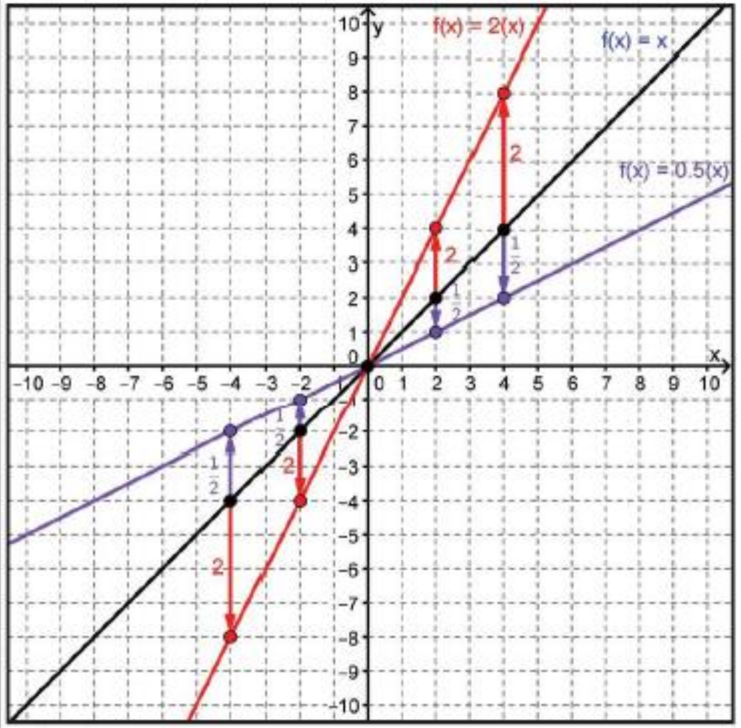
Linear Functions

- For a linear function, the general form is $f(x) = a(bx - c) + d$, where a, b, c, and d are real numbers.
- The linear parent function is $f(x) = x$
- The full family of linear functions is generated by applying transformations to the linear parent function
- Transformations are applied using parameters that are multiplied or added to the independent variable in the functional relationship

Changes in a

- The parameter a influences the vertical stretch or compression of the graph of the line.
- If $|a| > 1$, then the y-values are multiplied by a factor of a to vertically stretch the graph
- If $0 < |a| < 1$, then the y-values are multiplied by a factor of a to vertically compress the graph

Changes in a

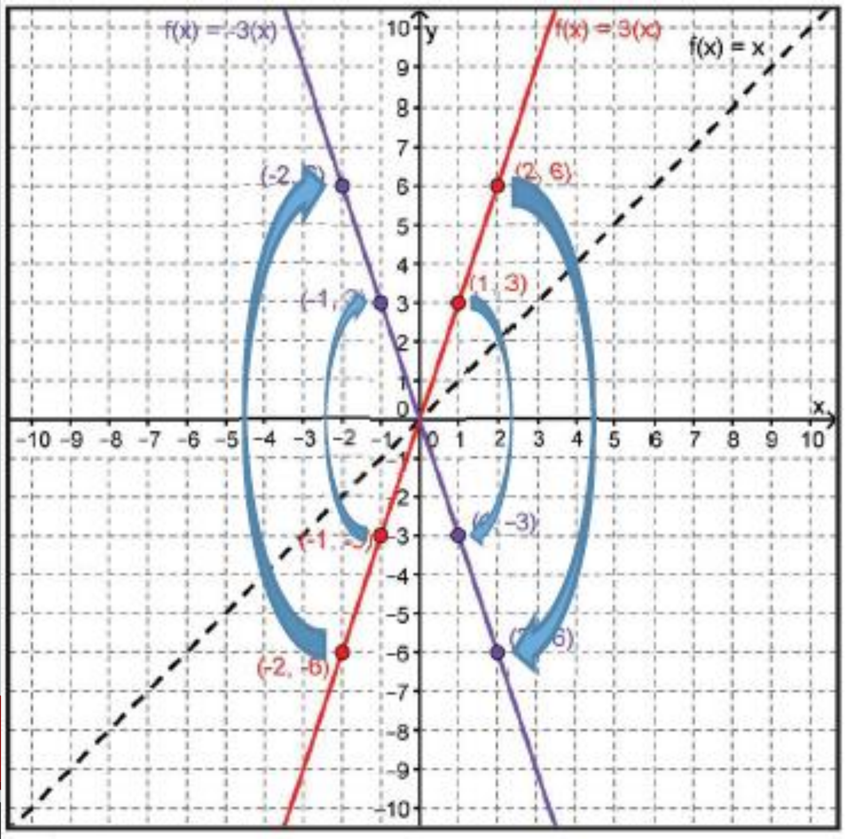


x	$f(x) = x$	$f(x) = 2(x)$	$f(x) = 0.5(x)$
−4	−4	−8	−2
−2	−2	−4	−1
0	0	0	0
2	2	4	1
4	4	8	2

× 2 × 0.5

Changes in *a*

- If $a < 0$, then the line will be reflected across the x-axis



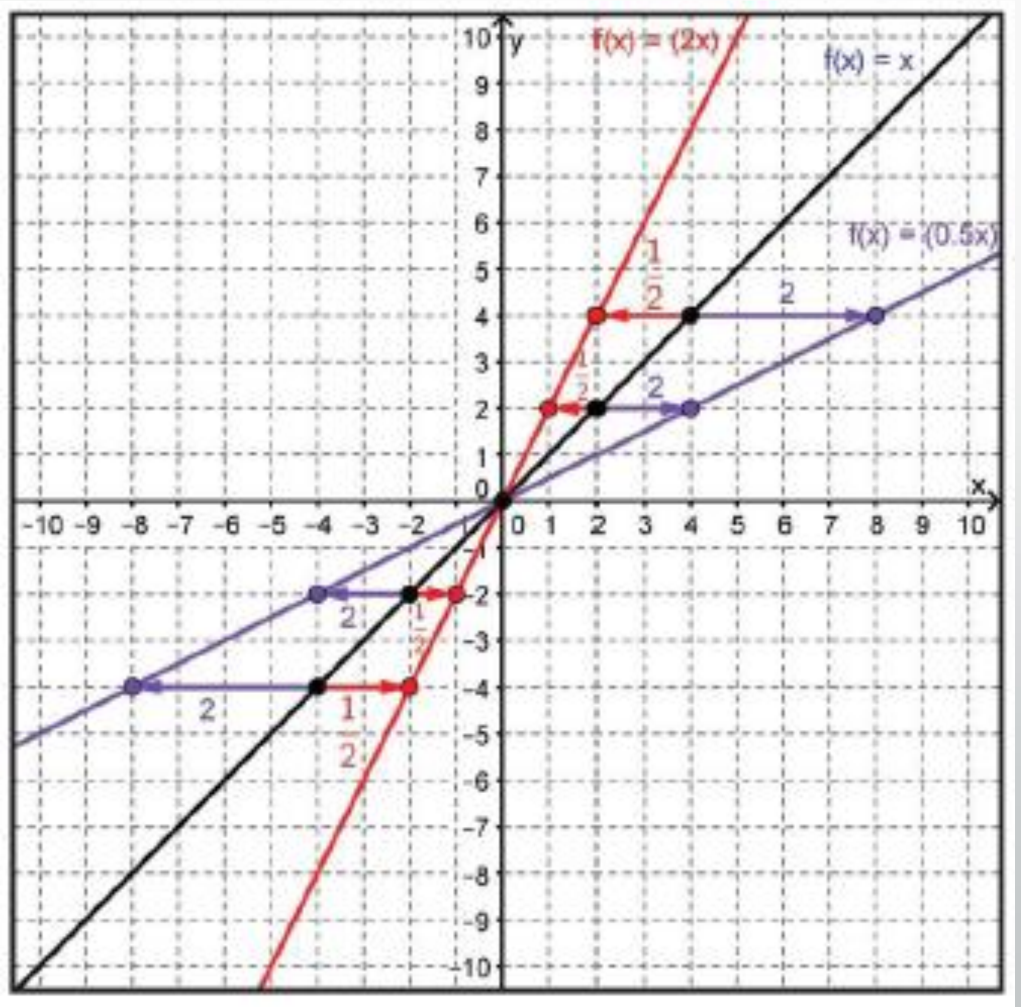
x	$f(x) = 3(x)$	$f(x) = -3(x)$
–2	–6	6
–1	–3	3
0	0	0
1	3	–3
2	6	–6

× –1

Changes in b

- The parameter b influences the horizontal stretch or compression of the graph of the line.
- If $|b| > 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally compress the graph
- If $0 < |b| < 1$, then the x-values are multiplied by a factor of $\frac{1}{|b|}$ to horizontally stretch the graph

Changes in *b*



Changes in *b*

X-values are multiplied by $\frac{1}{2}$ in order to generate the same y-value. This multiplication results in a horizontal compression of the graph.

x	$f(x) = x$
−4	−4
−2	−2
0	0
2	2
4	4

x	$f(x) = (2x)$
−2	−4
−1	−2
0	0
1	2
2	4

Changes in *b*

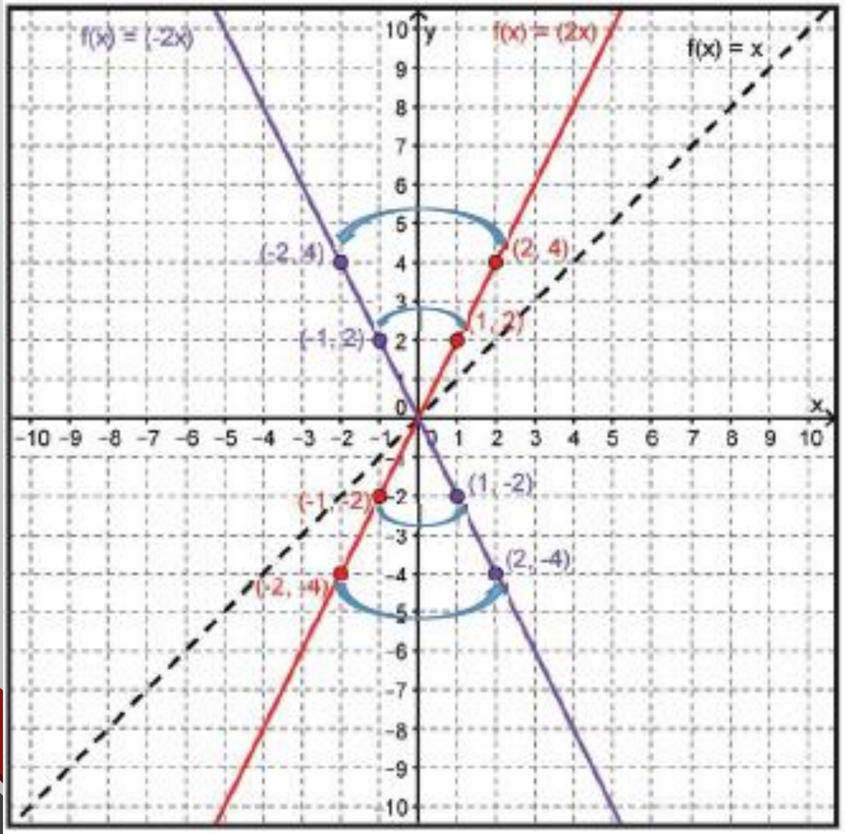
X-values are multiplied by 2 in order to generate the same y-value. This multiplication results in a horizontal stretch of the graph.

x	$f(x) = x$
-4	-4
-2	-2
0	0
2	2
4	4

x	$f(x) = (0.5x)$
-8	-4
-4	-2
0	0
4	2
8	4

Changes in *b*

- If $b < 0$, then the line will be reflected across the y-axis

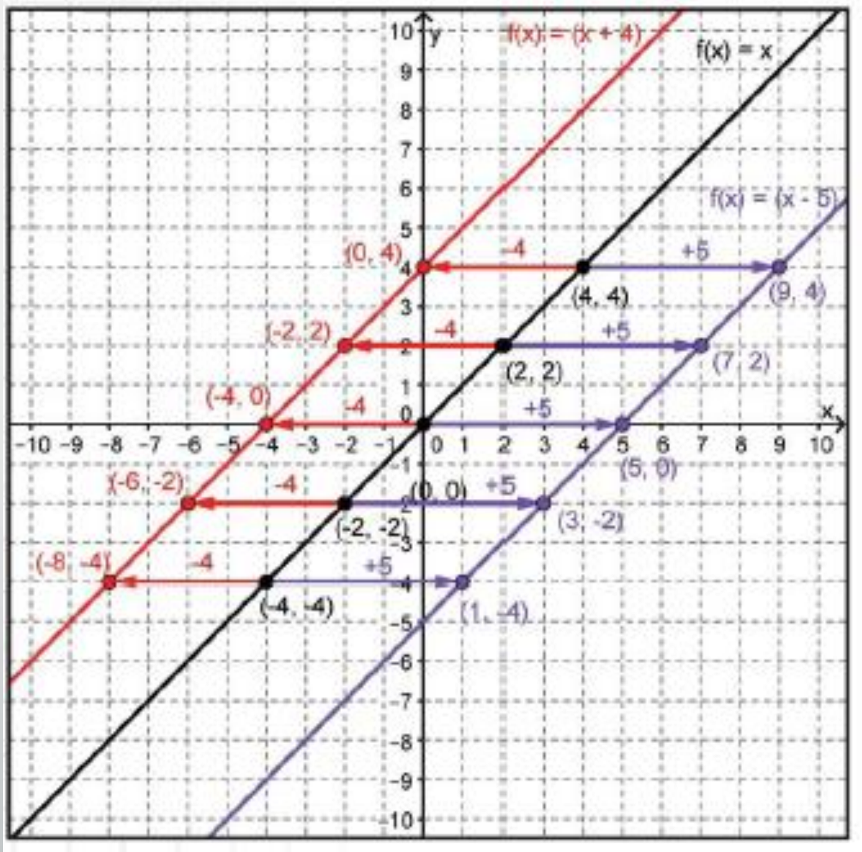


x	$f(x) = (2x)$	$f(x) = (-2x)$
-2	-4	4
-1	-2	2
0	0	0
1	2	-2
2	4	-4

Changes in *c*

- The parameter *c*, like *b*, influences the horizontal translation of the graph of the line.
- Note that in the general form, the sign in front of the *c* is negative. This means that when reading the value of *c* from the equation, you should read the opposite sign from what is given in the equation.
- If $c > 0$, then the graph will translate $|\frac{c}{b}|$ to the right.
- If $c < 0$, then the graph will translate $|\frac{c}{b}|$ to the left.

Changes in *c*



Changes in *c*

X-values are increased by 5 in order to generate the same y-value. This addition results in a horizontal translation of the graph to the right.

x	$f(x) = x$	x	$f(x) = (x - 5)$
−4	−4	1	−4
−2	−2	3	−2
0	0	5	0
2	2	7	2
4	4	9	4

Changes in *c*

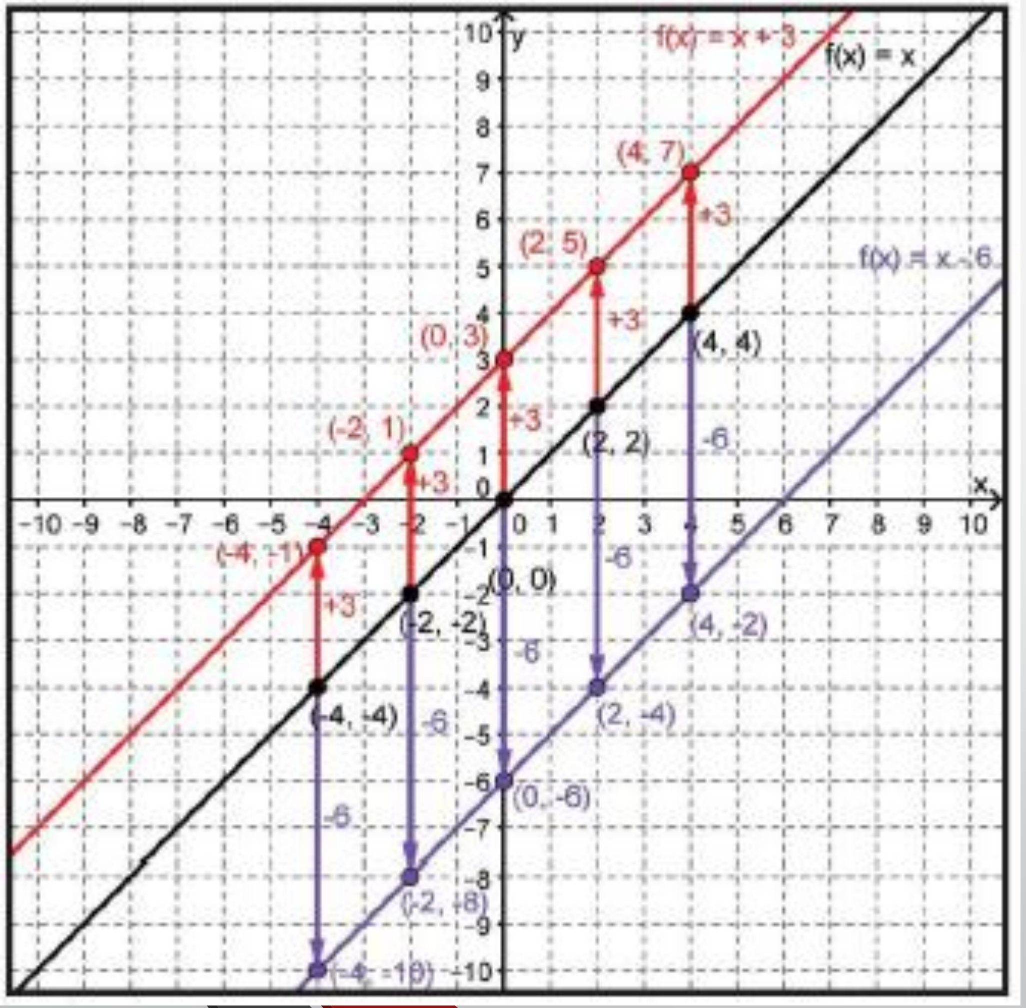
X-values are decreased by 4 in order to generate the same y-value. This subtraction results in a horizontal translation of the graph to the left.

x	$f(x) = x$	x	$f(x) = (x + 4)$
-4	-4	-8	-4
-2	-2	-6	-2
0	0	-4	0
2	2	-2	2
4	4	0	4

Changes in d

- The parameter d influences the vertical translation of the graph of the line.
- If $d > 0$, then the graph of the line will translate $|d|$ units up.
- If $d < 0$, then the graph of the line will translate $|d|$ units down.

Changes in *d*



x	$f(x) = x$	$f(x) = x - 6$	$f(x) = x + 3$
-4	-4	-10	-1
-2	-2	-8	1
0	0	-6	3
2	2	-4	5
4	4	-2	7

−6 +3

X- and Y-intercepts

- If the equation of the line is in the general form, y = a(bx – c) + d then:
  - the x-intercept becomes ($\frac{ac-d}{ab}$, o)
  - the y-intercept becomes (o, -ac + d)

Set Builder Notation

- The domain of a linear function is all real numbers. Using set builder notation the domain of a linear function is written as $\{x \mid x \in \mathbb{R}\}$
- The range of a linear function is all real numbers. Using set builder notation the domain of a linear function is written as $\{f(x) \mid f(x) \in \mathbb{R}\}$

Examples

- What transformations of the linear parent function, $f(x) = x$, will result in the graph of the linear function $g(x) = -3(0.5x + 4) + \frac{2}{5}$?

Examples

- Step 1: Rewrite the equation of g(x) in general form to determine the values of the parameters a, b, c, and d.
  - $g(x) = -3(0.5x + 4) + \frac{2}{5}$
  - $-3(0.5x - (-4) + \frac{2}{5}$
- So, a = -3, b = 0.5, c = -4, and d = $\frac{2}{5}$

Examples

- Step 2: Use the values of the parameters to describe the transformations of the linear parent function f(x) that are necessary to produce g(x).
- a = -3; so |a| > 1, then the y-values are multiplied by a factor of -3 to vertically stretch the graph and because *a* is negative, the graph is reflected over the x-axis
- b = 0.5; so 0 < $|b|$ < 1, then the x-values are multiplied by a factor of $\frac{1}{|0.5|}$ = 2 to horizontally stretch the graph
- c = -4, so *c* < 0, then the graph will translate $\left|\frac{-4}{0.5}\right|$ = 8 to the left
- d = $\frac{2}{5}$, so *d* > 0, then the graph of the line will translate $\left|\frac{2}{5}\right|$ units up

Examples

- What transformations of the linear parent function, f(x) = x, will result in the graph of the linear function g(x) = $\frac{1}{4}$(-6x - 5) + 1?

Examples

- Step 1: Rewrite the equation of g(x) in general form to determine the values of the parameters a, b, c, and d.
    - So, a = $\frac{1}{4}$, b = -6, c = 5, and d = 1

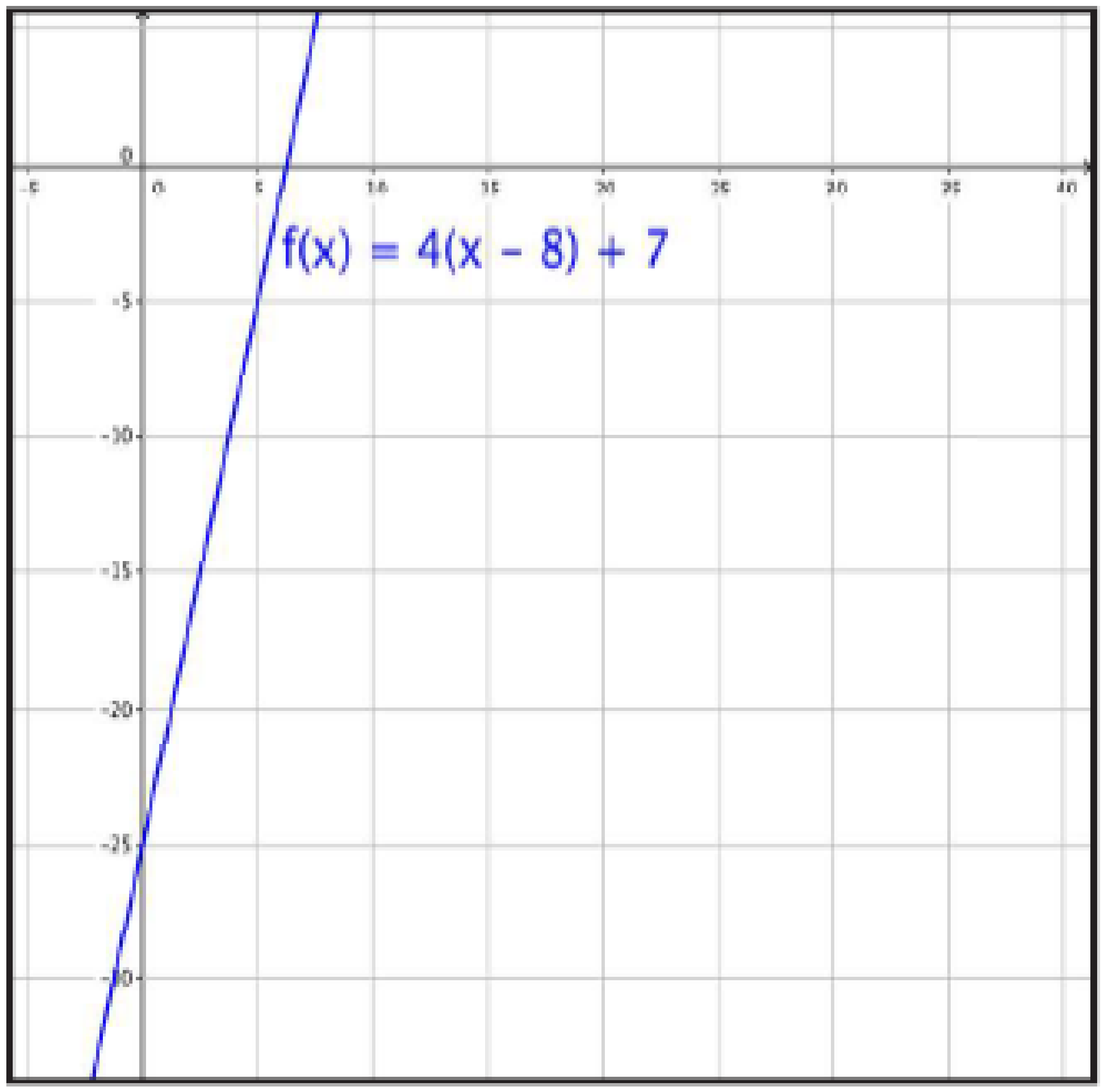
Examples

- Step 2: Use the values of the parameters to describe the transformations of the linear parent function f(x) that are necessary to produce g(x).

- a = $\frac{1}{4}$; 0 < $|a|$ < 1, then the y-values are multiplied by a factor of $\frac{1}{4}$ to vertically compress the graph
- b = -6; so $|b|$ > 1, then the x-values are multiplied by a factor of $\frac{1}{|-6|} = \frac{1}{6}$ to horizontally compress the graph
- c = 5, so c < 0, then the graph will translate $\left|\frac{5}{-6}\right| = \frac{5}{6}$ to the right
- d =1, so d > 0, then the graph of the line will translate $|1|$ units up

Examples

Identify the domain, range, x-intercept and y-intercept of the linear function described by the equation and graph shown. Write the domain and range as inequalities and in set builder notation.



Examples

- Step 1: Determine the domain and range

- Since it is a linear function, the domain and range are both all real numbers
  - Domain: inequality, $-\infty < x < \infty$; set builder, $\{x \mid x \in \mathbb{R}\}$
  - Range: inequality, $-\infty < y < \infty$; set builder, $\{f(x) \mid f(x) \in \mathbb{R}\}$

Examples

- Step 2: Determine the x-intercept
  - the x-intercept becomes ($\frac{ac-d}{ab}$, 0)
    - a = 4, b = 1, c = 8, d = 7
      - ($\frac{4*8-7}{4*1}$, 0)
      - ($\frac{32-7}{4}$, 0)
    - ($\frac{25}{4}$, 0) or (6.25, 0)

Examples

- Step 3: Determine the y-intercept
  - the y-intercept becomes (0, -ac + d)
    - a = 4, b = 1, c = 8, d = 7
      - (0, -4*8 + 7)
        - (0, -32 + 7)
          - (0, -25)