

Transforming and Analyzing Cubic Functions

For questions 1-8, describe the transformation of the cubic parent function, $f(x) = x^3$, that will result in the graph of the cubic function given.

2. $g(x) = -3(x)^3 + 4$

ANSWER:

$|a| > 1$, so the graph is vertically stretched by a factor of 3

$a < 0$, so graph is reflected over x-axis

$d = 4$, so horizontal shift 4 units to right

4. $g(x) = (-\frac{1}{4}x + 2)^3 + 5$

ANSWER:

$$g(x) = (-\frac{1}{4}x - (-2))^3 + 5$$

$b = -\frac{1}{4}$, so the graph is horizontally stretched by a factor of $\frac{1}{|-\frac{1}{4}|} = 4$

$b < 0$, so graph is reflected over y-axis

$c = -2$, so the graph will translate $|\frac{-2}{-\frac{1}{4}}| = 8$ units to the left

$d = 5$, so graph will translate 5 units up

6. $g(x) = \frac{1}{2}(4x + 3)^3 - 2$

ANSWER:

$$g(x) = \frac{1}{2}(4x - (-3))^3 - 2$$

$0 < a < 1$, so the graph is vertically compressed by a factor of $\frac{1}{2}$

$b = 4$, so the graph is horizontally compressed by a factor of $\frac{1}{|4|} = \frac{1}{4}$

$c = -3$, so the graph will translate $\frac{3}{4}$ units to the left

$d = -2$, so the graph will translate 2 units down

8. $g(x) = -(-6x + 5)^3 - 3$

ANSWER:

$$g(x) = -(-6x - (-5))^3 - 3$$

$a < 0$, so the graph is reflected over the x-axis

$b = -6$, so the graph is horizontally compressed by a factor of $\frac{1}{|-6|} = \frac{1}{6}$

$b < 0$, so the graph is reflected over the y-axis

$c = -5$, so the graph will translate $|\frac{-5}{-6}| = \frac{5}{6}$

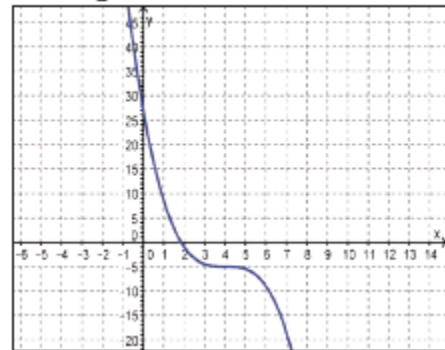
to the left

$d = -3$, so the graph will translate 3 units down

For questions 9-11, identify the domain, range, x-intercept, and y-intercept of each cubic function. Write the domain and range as an inequality and in set builder notation.

10.

$$f(x) = -\frac{1}{2}(x - 4)^3 - 5$$



SOLUTION:

Domain and range for a cubic function are both *All Real Numbers*

The x-intercept is found using a calculator: (1.85, 0)

The y-intercept is found when $x = 0$:

$$f(x) = -\frac{1}{2}(x - 4)^3 - 5 \rightarrow f(x) = -\frac{1}{2}(0 - 4)^3 - 5$$

$$f(x) = -\frac{1}{2}(-4)^3 - 5 \rightarrow f(x) = -\frac{1}{2}(-64) - 5$$

$$f(x) = 32 - 5 = 27 \rightarrow (0, 27)$$

ANSWER:

Domain: $-\infty < x < \infty$; $\{x | x \in \mathbb{R}\}$

Range: $-\infty < y < \infty$; $\{y | y \in \mathbb{R}\}$

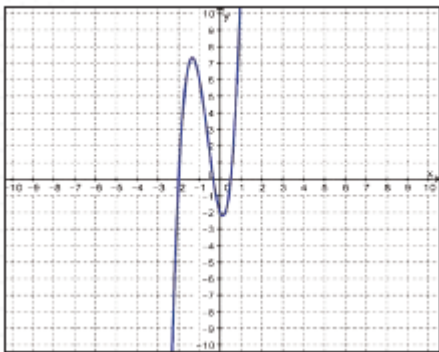
x-intercept: (1.85, 0)

y-intercept: (0, 27)

For questions 12 - 14, identify the domain, range, relative minimum, relative maximum, x-intercept, and y-intercept of the cubic function described by the equation and the graph. Write the domain and range as intervals. Use graphing technology to determine the relative minimum, relative maximum, and the x-intercepts.

12.

$$f(x) = (x + 2)(2x - 1)(3x + 1)$$



ANSWER:

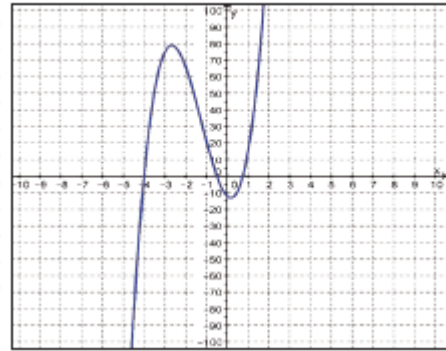
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

R. Max: (-1.35, 7.34) R. Min: (.12, -2.19)

x-int: (-2, 0), (-.33, 0), (.5, 0) y-int: (0, -2)

14.

$$f(x) = (4x - 3)(2x + 1)(x + 4)$$



ANSWER:

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

R. Max: (-1.35, 7.34) R. Min: (.12, -2.19)

x-int: (-4, 0), (-.5, 0), (.75, 0) y-int: (0, -12)