

Transforming and Analyzing Quadratic Functions

For questions 2-7, describe the transformation of the quadratic parent function, $f(x) = x^2$, that will result in the graph of the quadratic function given.

2. $g(x) = 2(x - 3)^2$

ANSWER:

$|a| > 1$, so the graph is vertically stretched by a factor of 2

$c = 3$, so horizontal shift 3 units to right

4. $g(x) = (4x - 7)^2$

ANSWER:

$b = 4$, so the graph is horizontally compressed by a factor of $\frac{1}{|4|} = \frac{1}{4}$

$c = 7$, so the graph will translate $|\frac{7}{4}| = \frac{7}{4}$ or 1.75 units to the right

6. $g(x) = -3(x + 2)^2 + 6$

ANSWER:

$$g(x) = -3(x - (-2))^2 + 6$$

$a < 0$, so the graph is reflected over the x-axis

$|a| > 1$, so the graph is vertically stretched by a factor of 3

$c = -2$, so the graph will translate 2 to the left

$d = 6$, so the graph will translate 6 units up

8. The graph of $g(x)$ is produced by transforming the quadratic parent function, $f(x) = x^2$, by vertically stretching its graph by a factor of 3 and translating it 7.5 units upward. Determine the equation that represents $g(x)$.

ANSWER:

$$g(x) = 3(x)^2 + 7.5$$

For questions 10-12, identify the vertex, and determine whether it is a maximum or a minimum value.

10. $g(x) = -(x - 1.5)^2 - 4$

SOLUTION:

$a < 0$, so the graph opens down, therefore the vertex will be a maximum

the vertex is found at $(\frac{c}{b}, d)$

the vertex will be at (1.5, -4)

ANSWER:

Maximum value at (1.5, -4)

12. $g(x) = (2x - 5)^2 + 3$

$a > 0$, so the graph opens up, therefore the vertex will be a minimum

the vertex is found at $(\frac{c}{b}, d)$

the vertex will be at $(\frac{5}{2}, 3) = (2.5, 3)$

ANSWER:

Minimum value at $(\frac{5}{2}, 3)$ or (2.5, 3)

For questions 13-17, identify the domain, range, x-intercept, y-intercept and vertex of each quadratic function. Write the domain and range in three different ways: as an inequality, interval, and in set builder notation.

14.

x	$f(x) = -(x + 1)^2 + 9$
-5	-7
-4	0
-3	5
-2	8
-1	9
0	8
1	5
2	0

SOLUTION:

Since this is a quadratic function, the domain is *all real numbers*. The range, from the table, appears to be numbers less than 9.

There are going to be x-intercepts ($y = 0$) at $(-4, 0)$ and $(2, 0)$, and a y-intercept ($x = 0$) at $(0, 8)$

The vertex is at $(\frac{c}{b}, d)$; $(-1, 9)$

ANSWER:

Domain: $-\infty < x < \infty$; $(-\infty, \infty)$; $\{x | x \in \mathbb{R}\}$

Range: $-\infty < y \leq 9$; $(-\infty, 9]$; $\{y | y \leq 9\}$

x-intercept: $(-4, 0)$ and $(2, 0)$

y-intercept: $(0, 8)$

or $(-1.42, 0)$ and $(-4.58, 0)$

y-intercept: $(0, 13)$

vertex: $(-3, -5)$

16.

x	$g(x) = 2(x+3)^2 - 5$
-5	3
-4	-3
-3	-5
-2	-3
-1	3
0	13
1	27

SOLUTION:

Since this is a quadratic function, the domain is *all real numbers*. The range, from the table, appears to be numbers greater than -5.

There are going to be x-intercepts ($y = 0$) at

$$\left(\frac{c \pm \sqrt{(-d)}}{b}, 0\right); \left(\frac{-3 \pm \sqrt{(-(-5))}}{1}, 0\right) = \left(\frac{-3 \pm \sqrt{\frac{5}{2}}}{1}, 0\right) =$$

$$\left(-3 \pm \sqrt{\left(\frac{5}{2}\right)}, 0\right) = \left(-3 + \sqrt{\left(\frac{5}{2}\right)}, 0\right) \text{ and } \left(-3 - \sqrt{\left(\frac{5}{2}\right)}, 0\right)$$

The y-intercept ($x = 0$) at $(0, 13)$

The vertex is at $(\frac{c}{b}, d)$; $(-3, -5)$

ANSWER:

Domain: $-\infty < x < \infty$; $(-\infty, \infty)$; $\{x | x \in \mathbb{R}\}$

Range: $-5 \leq y < \infty$; $[-5, \infty)$; $\{y | y \geq -5\}$

x-intercept: $\left(-3 + \sqrt{\left(\frac{5}{2}\right)}, 0\right)$ and $\left(-3 - \sqrt{\left(\frac{5}{2}\right)}, 0\right)$