

Transforming and Analyzing Linear Functions

For questions 1-8, describe the transformation of the linear parent function, $f(x) = x$, that will result in the graph of the linear function given.

2. $g(x) = -2(x) + 5$

ANSWER:

a is negative, so the graph is reflected over the x -axis

$|2| > 1$, so the graph is vertically stretched by a factor of 2

$d = 5$, so the graph will translate 5 units up

4. $g(x) = (-\frac{1}{2}x + 3) + 7$

ANSWER:

$$g(x) = (-\frac{1}{2}x - (-3)) + 7$$

b is negative, so the graph is reflected over the y -axis

$b = -\frac{1}{2}$, so the graph is horizontally stretched by a factor of $\frac{1}{|\frac{1}{2}|} = 2$

$c = -3$, so the graph will translate $|\frac{3}{\frac{1}{2}}| = 6$ to the left

$d = 7$, so the graph will translate 7 units up

6. $g(x) = \frac{2}{3}(6x + 1) - 3$

ANSWER:

$$g(x) = \frac{2}{3}(6x - (-1)) - 3$$

$a = \frac{2}{3}$; $0 < \frac{2}{3} < 1$, so the graph is vertically compressed by a factor of $\frac{2}{3}$

$b = 6$, so the graph is horizontally compressed by a factor of $\frac{1}{|6|} = \frac{1}{6}$

$c = -1$, so the graph will translate $|\frac{1}{6}| = \frac{1}{6}$ to the left

$d = -3$, so the graph will translate 3 units down

8. $g(x) = -(-8x + 9) - 6$

ANSWER:

$$g(x) = -(-8x - (-9)) - 6$$

a is negative, so the graph is reflected over the x -axis

b is negative, so the graph is reflected over the y -axis

$b = -8$, so the graph is horizontally compressed by a factor of $\frac{1}{|-8|} = \frac{1}{8}$

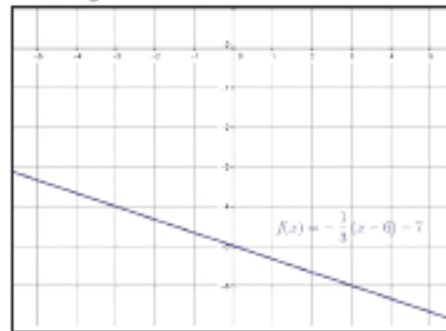
$c = -9$, so the graph will translate $|\frac{-9}{8}| = \frac{9}{8}$ to the left

$d = -6$, so the graph will translate 6 units down

For questions 9-12, identify the domain, range, x -intercept, and y -intercept of the linear function described by the equation and the graph. Write the domain and range as inequalities.

10.

$$f(x) = -\frac{1}{3}(x - 6) - 7$$



SOLUTION:

Since this is a linear function, the domain and range are both *all real numbers*.

$$a = \frac{1}{3}, b = 1, c = 6, \text{ and } d = -7$$

$$\begin{aligned} \text{The } x\text{-intercept is } & \left(\frac{ac-d}{ab}, 0\right); \left(\frac{-\frac{1}{3} \cdot 6 + 7}{-\frac{1}{3} \cdot 1}, 0\right) = \left(\frac{-2+7}{-\frac{1}{3}}, 0\right) = \\ & \left(\frac{5}{-\frac{1}{3}}, 0\right) = (-15, 0) \end{aligned}$$

$$\begin{aligned} \text{The } y\text{-intercept is } & (0, -ac + d); \left(0, -\left(-\frac{1}{3}\right) \cdot 6 - 7\right) = \\ & = (0, 2 - 7) = (0, -5) \end{aligned}$$

ANSWER:

Domain: $-\infty < x < \infty$

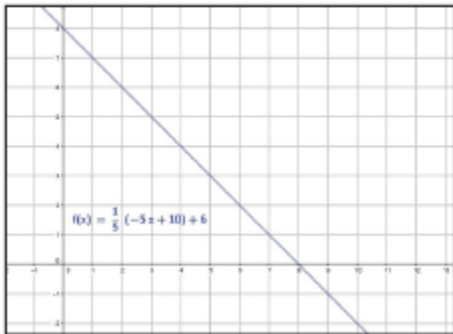
Range: $-\infty < y < \infty$

x-intercept: $(-15, 0)$

y-intercept: $(0, -5)$

12.

$$f(x) = \frac{1}{5}(-5x + 10) + 6$$



SOLUTION:

Since this is a linear function, the domain and range are both *all real numbers*.

$a = \frac{1}{5}$, $b = -5$, $c = -10$, and $d = 6$

The x-intercept is $(\frac{ac-d}{ab}, 0)$; $(\frac{\frac{1}{5}(-10)-6}{\frac{1}{5}*(-5)}, 0) = (\frac{-2-6}{-1}, 0) = (\frac{-8}{-1}, 0) = (8, 0)$

The y-intercept is $(0, -ac + d)$; $(0, -\frac{1}{5} * -10 + 6)$
 $= (0, 2 + 6) = (0, 8)$

ANSWER:

Domain: $-\infty < x < \infty$

Range: $-\infty < y < \infty$

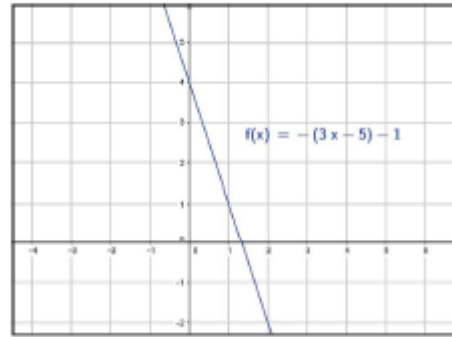
x-intercept: $(8, 0)$

y-intercept: $(0, 8)$

For questions 13-16, identify the domain, range, x-intercept, and y-intercept of the linear function described by the equation and the graph. Write the domain and range in set builder notation.

14.

$$f(x) = -(3x - 5) - 1$$



SOLUTION:

Since this is a linear function, the domain and range are both *all real numbers*.

$a = -1$, $b = 3$, $c = 5$, and $d = -1$

The x-intercept is $(\frac{ac-d}{ab}, 0)$; $(\frac{-1*5+1}{-1*3}, 0) = (\frac{-5+1}{-3}, 0) = (\frac{-4}{-3}, 0) = (\frac{4}{3}, 0)$

The y-intercept is $(0, -ac + d)$; $(0, -(-1)*5 - 1)$
 $= (0, 5 - 1) = (0, 4)$

ANSWER:

Domain: $\{x | x \in \mathbb{R}\}$

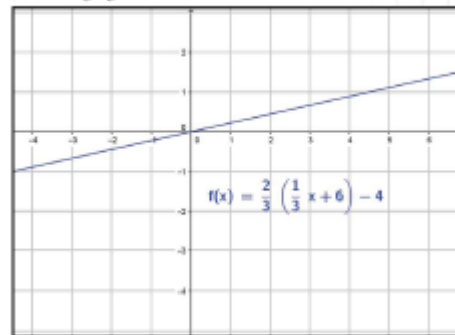
Range: $\{y | y \in \mathbb{R}\}$

x-intercept: $(\frac{4}{3}, 0)$

y-intercept: $(0, 4)$

16.

$$f(x) = \frac{2}{3}(\frac{1}{3}x + 6) - 4$$



SOLUTION:

Since this is a linear function, the domain and range are both *all real numbers*.

$$a = \frac{2}{3}, b = \frac{1}{3}, c = -6, \text{ and } d = -4$$

$$\text{The x-intercept is } \left(\frac{ac-d}{ab}, 0\right); \left(\frac{\frac{2}{3}*(-6)+4}{\frac{2}{3}*\frac{1}{3}}, 0\right) = \left(\frac{-4+4}{\frac{2}{9}}, 0\right) = \left(\frac{0}{\frac{2}{9}}, 0\right) = (0, 0)$$

$$\text{The y-intercept is } (0, -ac + d); \left(0, \frac{2}{3}*(-6) - 4\right) = (0, 4 - 4) = (0, 0)$$

ANSWER:

$$\text{Domain: } \{x | x \in \mathbb{R}\}$$

$$\text{Range: } \{y | y \in \mathbb{R}\}$$

$$\text{x-intercept: } (0, 0)$$

$$\text{y-intercept: } (0, 0)$$

For questions 17-20, identify the domain, range, x-intercept, and y-intercept of the linear function described by the equation and the graph. Write the domain and range as intervals.

18.

$$f(x) = \frac{1}{2}(-6x - 3)$$

x	f(x)
-4	10.5
-2	4.5
1	-4.5
3	-10.5
5	-16.5

SOLUTION:

Since this is a linear function, the domain and range are both *all real numbers*.

$$a = \frac{1}{2}, b = -6, c = 3, \text{ and } d = 0$$

$$\text{The x-intercept is } \left(\frac{ac-d}{ab}, 0\right); \left(\frac{\frac{1}{2}*3}{\frac{1}{2}*(-6)}, 0\right) = \left(\frac{\frac{3}{2}}{-3}, 0\right) = \left(\frac{-1}{2}, 0\right) = (-0.5, 0)$$

$$\text{The y-intercept is } (0, -ac + d); \left(0, -\frac{1}{2}*3\right) = \left(0, \frac{-3}{2}\right) = (0, -1.5)$$

ANSWER:

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (-\infty, \infty)$$

$$\text{x-intercept: } (-0.5, 0)$$

$$\text{y-intercept: } (0, -1.5)$$

20.

$$f(x) = -\frac{1}{3}\left(\frac{1}{4}x - 9\right) - 5$$

x	f(x)
-12	-3
-6	-2.5
-3	-2.25
3	-1.75
9	-1.25

SOLUTION:

Since this is a linear function, the domain and range are both *all real numbers*.

$$a = \frac{1}{3}, b = \frac{1}{4}, c = 9, \text{ and } d = -5$$

$$\text{The x-intercept is } \left(\frac{ac-d}{ab}, 0\right); \left(\frac{\frac{1}{3}*9+5}{\frac{1}{3}*\frac{1}{4}}, 0\right) = \left(\frac{-3+5}{\frac{1}{12}}, 0\right) = \left(\frac{2}{\frac{1}{12}}, 0\right) = (24, 0)$$

$$\text{The y-intercept is } (0, -ac + d); \left(0, -\left(\frac{1}{3}\right)*9 - 5\right) = (0, 3 - 5) = (0, -2)$$

ANSWER:

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (-\infty, \infty)$$

$$\text{x-intercept: } (24, 0)$$

$$\text{y-intercept: } (0, -2)$$