Modeling Cubic Functions

For questions 1-6, determine whether the set of data represents a linear, exponential, quadratic or cubic function.

2.

x	y
0	-6
1	1
2	16
3	39
4	70
5	109

SOLUTION:

$\Delta y = 7$ $\Delta^2 y = 8$

$\Delta y = 15$ $\Delta^2 y = 8$

$\Delta y = 23$ $\Delta^2 y = 8$

$\Delta y = 31$ $\Delta^2 y = 8$

$\Delta y = 39$

The second finite differences are all 8, so the data represents a quadratic function

ANSWER:

Quadratic function

4.

x	y
0	5
1	10
2	35
3	92
4	193
5	350

SOLUTION:

$\Delta y = 5$ $\Delta^2 y = 20$ $\Delta^3 y = 12$

$\Delta y = 25$ $\Delta^2 y = 32$ $\Delta^3 y = 12$

$\Delta y = 57$ $\Delta^2 y = 44$ $\Delta^3 y = 12$

$\Delta y = 101$ $\Delta^2 y = 56$

$\Delta y = 157$

The third finite differences are all 12, so the data represents a cubic function

ANSWER:

Cubic function

6.

x	y
0	-8
1	3
2	44
3	145
4	336
5	647

SOLUTION:

$\Delta y = 11$ $\Delta^2 y = 30$ $\Delta^3 y = 30$

$\Delta y = 41$ $\Delta^2 y = 60$ $\Delta^3 y = 30$

$\Delta y = 101$ $\Delta^2 y = 90$ $\Delta^3 y = 30$

$\Delta y = 191$ $\Delta^2 y = 120$

$\Delta y = 311$

The third finite differences are all 30, so the data represents a cubic function

ANSWER:

Cubic function

For questions 7-12, the data sets shown in the tables represent cubic functions. Use finite differences to determine the function that relates the variables.

8.

x	y
0	3
1	7
2	1
3	-27
4	-89
5	-197

SOLUTION:

$\Delta y = 4$ $\Delta^2 y = -10$ $\Delta^3 y = -12$

$\Delta y = -6$ $\Delta^2 y = -22$ $\Delta^3 y = -12$

$\Delta y = -28$ $\Delta^2 y = -34$ $\Delta^3 y = -12$

$\Delta y = -62$ $\Delta^2 y = -46$

$\Delta y = -108$

The third finite differences are all -12, so the data represents a cubic function

$\Delta^3 y = -12$, $6a = -12$; $a = -2$

$\Delta^2 y = -10$, $6a + 2b = -10$; $-12 + 2b = -10$; $2b = 2$; $b = 1$

$\Delta y = 4$, $a + b + c = 4$; $-2 + 1 + c = 4$; $c = 5$

y-int = d = 3

ANSWER:

$y = -2x^3 + 1x^2 + 5x + 3$

10.

x	y
0	-4
1	-9
2	-48
3	-157
4	-372
5	-729

SOLUTION:

$\Delta y = -5$ $\Delta^2 y = -34$ $\Delta^3 y = -36$

$\Delta y = -39$ $\Delta^2 y = -70$ $\Delta^3 y = -36$

$\Delta y = -109$ $\Delta^2 y = -106$ $\Delta^3 y = -36$

$\Delta y = -215$ $\Delta^2 y = -142$

$\Delta y = -357$

The third finite differences are all -36, so the data represents a cubic function

$\Delta^3 y = -36$, $6a = -36$; $a = -6$

$\Delta^2 y = -34$, $6a + 2b = -34$; $-36 + 2b = -34$; $2b = 2$; $b = 1$

$\Delta y = -5$, $a + b + c = -5$; $-6 + 1 + c = -5$; $c = 0$

y-int = d = -4

ANSWER:

$y = -6x^3 + x^2 - 4$

12.

x	y
0	-9
1	-3
2	39
3	153
4	375
5	741

SOLUTION:

$\Delta y = 6$ $\Delta^2 y = 36$ $\Delta^3 y = 36$

$\Delta y = 42$ $\Delta^2 y = 72$ $\Delta^3 y = 36$

$\Delta y = 114$ $\Delta^2 y = 108$ $\Delta^3 y = 36$

$\Delta y = 222$ $\Delta^2 y = 144$

$\Delta y = 366$

The third finite differences are all 36, so the data represents a cubic function

$\Delta^3 y = 36$, $6a = 36$; $a = 6$

$\Delta^2 y = 36$, $6a + 2b = 36$; $36 + 2b = 36$; $2b = 0$; $b = 0$

$\Delta y = 6$, $a + b + c = 6$; $6 + 0 + c = 6$; $c = 6$

y-int = d = x-9

ANSWER:

$y = 6x^3 - 9$

For questions 13-17, use the following information.

A box is created from a 20-inch by 24-inch rectangular piece of cardboard by cutting congruent squares from each corner. The squares are cut in 1-inch increments. The resulting sides are folded up and taped to form a rectangular prism (open box). The volume of the box is a function of the side length of the square removed from each corner. The table below relates the volume of the box to the side length of the square.

SIDE LENGTH x	VOLUME y
0	0
1	396
2	640
3	756
4	768
5	700
6	576
7	420
8	256
9	108

14. What side length of the square produces a tray with the greatest volume?

SOLUTION:

Look at the y-values for the largest number, then give the corresponding x-value.

ANSWER:

4 inches

For questions 18 – 22, use the scenario below.

An employee at a toy store is creating a display of soccer balls in the shape of a tetrahedron, or an equilateral triangle pyramid.

The table below shows the total number of soccer balls at each level of the display, with Level 1 being at the top of the display.

LEVEL, x	TOTAL NUMBER OF SOCCER BALLS, y
1	1
2	4
3	10
4	20
5	35
6	56

18. Write a function using finite differences that models the data in the table.

SOLUTION:

$\Delta y = 1$ $\Delta^2 y = 2$ $\Delta^3 y = 1$

$\Delta y = 3$ $\Delta^2 y = 3$ $\Delta^3 y = 1$

$\Delta y = 6$ $\Delta^2 y = 4$ $\Delta^3 y = 1$

$\Delta y = 10$ $\Delta^2 y = 5$ $\Delta^3 y = 1$

$\Delta y = 15$ $\Delta^2 y = 6$

$\Delta y = 21$

The third finite differences are all 1, so the data represents a cubic function

$\Delta^3 y = 1$, $6a = 1$; $a = 1/6$

$\Delta^2 y = 2$, $6a + 2b = 2$; $1 + 2b = 2$; $2b = 1$; $b = 1/2$

$\Delta y = 1$, $a + b + c = 1$; $1/6 + 1/2 + c = 1$; $c = 1/3$

y-int = d = 0

ANSWER:

$y = \frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{1}{3}x$

20. What does the range (y-values) of the function represent in the situation?

ANSWER:

The total number of soccer balls

22. How many soccer balls would be needed to build a display 10 levels high?

SOLUTION:

Make a table of the function, $y = \frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{1}{3}x$ and look at where the x-value is 10.

ANSWER:

220 soccer balls