

Modeling with Linear Functions

For the following sets of data, calculate the average finite difference, and use that to determine the slope of a linear function that could model the data.

1.

x	y
1	15.3
2	25.3
3	35.2
4	45.4
5	55.2
6	65.3

SOLUTION:

$$\Delta x = 1; \Delta y = +10, +9.9, +10.2, +9.8, +10.1$$

$$\text{Avg } \Delta y = \frac{10 + 9.9 + 10.2 + 9.8 + 10.1}{6} = 10$$

$$\frac{\Delta y}{\Delta x} = \frac{10}{1} = 10$$

ANSWER:

10

2.

x	0	1	2	3	4	5	6
y	50	47.2	44.3	41.5	38.5	35.6	32.5

SOLUTION:

$$\Delta x = 1; \Delta y = -2.8, -2.9, -2.8, -3, -2.9, -3.1$$

$$\text{Avg } \Delta y = \frac{-2.8 - 2.9 - 2.8 - 3 - 2.9 - 3.1}{6} = -2.9$$

$$\frac{\Delta y}{\Delta x} = \frac{-2.9}{1} = -2.9$$

ANSWER:

-2.9

3.

x	1	2	3	4	5
y	14.25	14.05	15.35	16	16.55

SOLUTION:

$$\Delta x = 1; \Delta y = -0.2, +1.3, +0.65, +0.55$$

$$\text{Avg } \Delta y = \frac{-0.2 + 1.3 + 0.65 + 0.55}{4} = .575$$

$$\frac{\Delta y}{\Delta x} = \frac{.575}{1} = .575$$

ANSWER:

.575

For problems 4 – 6, determine a linear function to model the situation.

4. Madeleine has a gift card to her favorite coffee shop. The table below shows how much is remaining on the gift card after each purchase at the coffee shop.

PURCHASES	0	1	2	3	4	5
BALANCE (DOLLARS)	40	35.68	31.22	26.97	22.65	18.40

SOLUTION:

$$\Delta x = 1; \Delta y = -4.32, -4.46, -4.25, -4.32, -4.25$$

$$\text{Avg } \Delta y = \frac{-4.32 - 4.46 - 4.25 - 4.32 - 4.25}{5} = -4.32$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-4.32}{1} = -4.32$$

$$b = 40$$

$$y = -4.32x + 40$$

ANSWER:

$$y = -4.32x + 40$$

5. Gus records the mileage on his car so he can determine his average mileage per month. Below are some of his collected data.

TIME (MONTHS)	1	2	3	4	5	6
MILEAGE (MILES)	11,540	12,482	13,570	14,670	15,682	16,757

SOLUTION:

$$\Delta x = 1; \Delta y = +942, +1088, +1100, +1012, +1075$$

$$\text{Avg } \Delta y = \frac{942 + 1088 + 1100 + 1012 + 1075}{5} = 1043.4$$

$$m = \frac{\Delta y}{\Delta x} = \frac{1043.4}{1} = 1043.4$$

$$b = 11,540 - 1043.4 = 10,496.6$$

$$y = 1043.4x + 10,496.6$$

ANSWER:

$$y = 1043.4x + 10,496.6$$

6. David is purchasing apps for his cell phone. The table below shows how his total cost changes with each app that he selects.

NUMBER OF APPS PURCHASED	1	2	3	4	5
COST (DOLLARS)	1.25	2.50	3.75	5.00	6.25

SOLUTION:

$$\Delta x = 1; \Delta y = +1.25, +1.25, +1.25, +1.25$$

$$\text{Avg } \Delta y = \frac{1.25 + 1.25 + 1.25 + 1.25}{4} = 1.25$$

$$m = \frac{\Delta y}{\Delta x} = \frac{1.25}{1} = 1.25$$

$$b = 1.25 - 1.25 = 0$$

$$y = 1.25x + 0$$

ANSWER:

$$y = 1.25x$$

Use the following situation to answer problems 7 – 10.

Charlie is measuring his little brother's height throughout the year to see how much he grows. The table below shows how his height changes during the first 5 months.

TIME (MONTHS)	0	1	2	3	4	5
HEIGHT (INCHES)	54	54.20	54.45	54.85	55.10	55.25

7. Write a function rule to model the situation.

SOLUTION:

$$\Delta x = 1; \Delta y = +0.20, +0.25, +0.40, +0.25, +0.15$$

$$\text{Avg } \Delta y = \frac{0.20 + 0.25 + 0.40 + 0.25 + 0.15}{5} = 0.25$$

$$m = \frac{\Delta y}{\Delta x} = \frac{0.25}{1} = 0.25$$

$$b = 54$$

$$y = 0.25x + 54$$

ANSWER:

$$y = 0.25x + 54$$

8. What do the slope and y-intercept from your function rule mean in the context of this situation?

ANSWER:

The slope tells that he grows 0.25 in per month and the y-intercept tells us that he started at 54 in

9. Use your model to predict the height of Charlie's brother after a year.

SOLUTION:

$$y = 0.25x + 54$$

$$y = 0.25(12) + 54$$

$$y = 3 + 54$$

$$y = 57 \text{ in}$$

ANSWER:

$$y = 57 \text{ in}$$

10. In what month will his height be approximately 56 inches?

SOLUTION:

$$y = 0.25x + 54$$

$$56 = 0.25x + 54$$

$$56 - 54 = 0.25x + 54 - 54$$

$$2 = 0.25x$$

$$\frac{2}{0.25} = \frac{0.25x}{0.25}$$

$$8 = x$$

ANSWER:

$x = 8$; the 8th month

Use the following situation to answer problems 11 – 13.

Jeff noticed that the nutrition information on his box of cereal states that there are 14 servings in the cereal box. He decided to put their claim to the test. He recorded the weight of the remaining cereal after each serving, as shown in the table below.

NUMBER OF SERVINGS	0	1	2	3	4	5
WEIGHT OF REMAINING CEREAL (OUNCES)	14	12.7	11.6	10.7	9.5	8.4

11. Write a function rule that models the situation.

SOLUTION:

$$\Delta x = 1; \Delta y = -1.3, -1.1, -0.9, -1.2, -1.1$$

$$\text{Avg } \Delta y = \frac{-1.3 - 1.1 - 0.9 - 1.2 - 1.1}{5} = -1.12$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-1.12}{1} = -1.12$$

$$b = 14$$

$$y = -1.12x + 14$$

ANSWER:

$$y = -1.12x + 14$$

12. What do the slope and y-intercept from your function rule mean in the context of this situation?

ANSWER:

The slope tells that he uses 1.12 ounces per serving and the y-intercept tells that he started with 14 ounces of cereal

13. Was Jeff able to confirm the claim on the cereal box by eating 14 servings? Explain your answer.

SOLUTION:

$$y = -1.12x + 14$$

$$0 = -1.12x + 14$$

$$0 - 14 = -1.12x + 14 - 14$$

$$-14 = -1.12x$$

$$\frac{-14}{-1.12} = \frac{-1.12x}{-1.12}$$

$$12.5 = x$$

$$x = 12.5$$

ANSWER:

$x = 12.5$; he will finish in 12.5 servings, not 14. But we don't know how many ounces he used. They only told us how many were left.

Use the following situation to answer problems 14 – 17.

Bob is tracking a hurricane moving toward the coast of Florida. The table shows its distance from land over time.

TIME (HOURS)	0	1	2	3	4
DISTANCE (MILES)	704	684	663.7	644.2	624.4

14. Write a function rule that models the situation.

SOLUTION:

$$\Delta x = 1; \Delta y = -20, -20.3, -19.5, -19.8$$

$$\text{Avg } \Delta y = \frac{-20 - 20.3 - 19.5 - 19.8}{4} = -19.9$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-19.9}{1} = -19.9$$

$$b = 704$$

$$y = -19.9x + 704$$

ANSWER:

$$y = -19.9x + 704$$

15. What do the slope and y-intercept from your function rule mean in the context of this situation?

ANSWER:

The slope tells that the hurricane is getting 19.9 miles closer to land every hour and the y-intercept tells that it started 704 miles away from land

16. About how far will the hurricane be from land after 24 hours?

SOLUTION:

$$y = -19.9x + 704$$

$$y = -19.9(24) + 704$$

$$y = -477.6 + 704$$

$$y = 226.4$$

ANSWER:

$$y = 226.4 \text{ miles}$$

17. Approximately when will the hurricane make landfall?

SOLUTION:

$$y = -19.9x + 704$$

$$0 = -19.9x + 704$$

$$0 - 704 = -19.9x + 704 - 704$$

$$-704 = -19.9x$$

$$\frac{-704}{-19.9} = \frac{-19.9x}{-19.9}$$

$$35.38 = x$$

$$x = 35.38$$

ANSWER:

Shortly after 35 hours

Use the following situation to answer problems 18 – 19.

Maddie is running a 10-K (10 kilometer) race. She wears an electronic chip that tracks her progress throughout the race. She runs at a fairly steady pace throughout the race, as shown in her chip data below.

DISTANCE (KILOMETERS)	0	1	2	3	4	5
TIME (MINUTES)	0	4.1	7.9	11.8	15.5	19

18. Write a function rule that models the situation.

SOLUTION:

$$\Delta x = 1; \Delta y = +3.8, +3.9, +3.7, +3.5$$

$$\text{Avg } \Delta y = \frac{4.1 + 3.8 + 3.9 + 3.7 + 3.5}{5} = 3.8$$

$$m = \frac{\Delta y}{\Delta x} = \frac{3.8}{1} = 3.8$$

$$b = 0$$

$$y = 3.8x + 0$$

ANSWER:

$$y = 3.8x$$

19. If Maddie continues at this rate, will she beat her previous best time of 37.5 minutes? Explain your answer.

SOLUTION:

$$y = 3.8x$$

$$y = 3.8(10)$$

$$y = 38$$

ANSWER:

$y = 38$ minutes; no, she will not beat her previous time, she will be 0.5 minutes slower at this rate

SOLUTION:

$$\Delta x = 1; \Delta y = +9.3, +10, +11.43, +12.5, +13.1$$

$$\text{Avg } \Delta y = \frac{9.3 + 10 + 11.43 + 12.5 + 13.1}{5} = 11.3$$

$$m = \frac{\Delta y}{\Delta x} = \frac{11.3}{1} = 11.3$$

$$b = 0$$

$$y = 11.3x + 0$$

ANSWER:

Her new equation looks like $y = 11.3x$; it's close to the previous one, but now it goes through the point $(0, 0)$.

Use the following situation to answer problems 20 – 21.

Nikki has a job as a waitress where she gets an hourly wage plus tips. The table below shows her total earnings for working one weekend.

TIME WORKED (HOURS)	1	2	3	4	5
TOTAL EARNED (DOLLARS)	9.3	19.3	30.73	43.23	56.33

Nikki calculates that the function $f(x) = 11.8x - 2.5$ models her earnings over time. She understands that the slope of 11.8 means she earned an average of about \$11.80 per hour. However, she is uncertain about why she has a negative y-intercept in her function equation, since she didn't earn -\$2.50 for working 0 hours.

20. Is her equation correct? Explain why or why not.

SOLUTION:

$$b = 9.3 - 11.8 = -2.5$$

ANSWER:

Yes, her equation is correct; it is not going to be exact, it is an approximation

21. Since she earned \$0 for working zero hours, she now decides to include the point $(0, 0)$ in her data set. How will this affect her function equation to model the situation?