

Study Guide and Intervention

Writing Quadratic Functions

Example 1 Write a function rule.

Write an equation for the function.

x	y
1	1
2	9
3	23
4	43
5	69

Solution

Step 1 Determine the finite differences.

$\Delta x = 1 - 0 = 1$	x	y	$\Delta y = 9 - 1 = 8$
$\Delta x = 2 - 1 = 1$	1	1	$\Delta y = 23 - 9 = 14$
$\Delta x = 3 - 2 = 1$	2	9	$\Delta y = 43 - 23 = 20$
$\Delta x = 4 - 3 = 1$	3	23	$\Delta y = 69 - 43 = 26$
	4	43	
	5	69	

Step 2 Determine whether or not the differences are constant.

$\Delta x = 1$, so they are constant
 Δy is not constant

Step 3 Determine whether or not the second finite differences in successive y -values are constant.

$$\begin{array}{l} \Delta y = 9 - 1 = 8 \\ \Delta y = 23 - 9 = 14 \\ \Delta y = 43 - 23 = 20 \\ \Delta y = 69 - 43 = 26 \end{array} \quad \begin{array}{l} \Delta^2 y = 14 - 8 = 6 \\ \Delta^2 y = 20 - 14 = 6 \\ \Delta^2 y = 26 - 20 = 6 \end{array}$$

The second differences are all equal to 6 and are constant.

Step 4 Calculate a , b , and c for the quadratic function $f(x) = ax^2 + bx + c$

The second finite difference is $2a$, so $2a = 6$ and $a = 3$.

The first difference between $x = 0$ and $x = 1$ is equal to $a + b$, so $a + b = 2$. Since $a = 3$, b must equal -1 .

Since the second finite difference is 6, and the first finite difference between $x = 0$ and $x = 1$ is found by subtracting 6 from the first finite difference between $x = 1$ and $x = 2$, 8, then the finite difference becomes 2 ($8 - 6 = 2$). Therefore, $1 - c = 2$; $c = -1$.

Step 5 Write the function with the values for a , b , and c .

$$f(x) = 3x^2 - x - 1$$

Exercises

For questions 1-3, use finite differences to determine if the data sets represent linear, exponential, quadratic, or other type of function.

1.

x	$y = f(x)$
1	5
2	11
3	21
4	35
5	53

2.

x	$y = f(x)$
1	5
2	9
3	16
4	29
5	52

3.

x	$y = f(x)$
1	5
2	12
3	31
4	68
5	129

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Writing Quadratic Functions (cont.)

Exercises

For questions 4-5, the data sets shown in the tables represent quadratic functions. Use finite differences to determine the values a , b , and c and then write the function in standard form.

4.

x	$y = f(x)$
0	7
1	10
2	19
3	34

5.

x	$y = f(x)$
0	-1
1	5
2	19
3	41

For questions 6-8, the data sets shown in the tables represent quadratic functions. Use finite differences to determine $f(0)$, the values a , b , and c and then write the function in standard form.

6.

x	$y = f(x)$
0	?
1	-1
2	5
3	13
4	23

7.

x	$y = f(x)$
0	?
1	3
2	16
3	41
4	78

8.

x	$y = f(x)$
0	?
1	-9
2	-8
3	-1
4	12

9. Using the pattern of second differences, determine the number of dots that would appear in the 5th figure.

FIGURE NUMBER, n	1	2	3	4	5	n
NUMBER OF DOTS, $D(n)$	1	6	15	28		

10. Write a quadratic function to represent the relationship between n and $D(n)$.