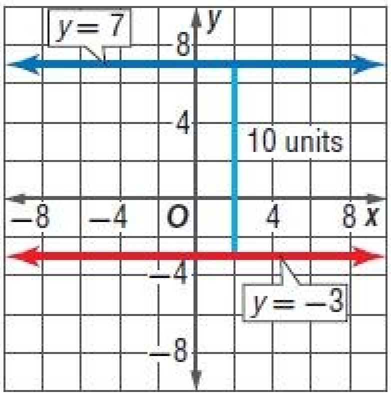
Find the distance between each pair of parallel lines with the given equations.

8. $y = 7$
$y = -3$

SOLUTION:



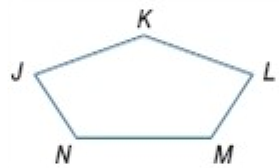
The two lines have the coefficient of x, zero. So, the slopes are zero. Therefore, the lines are horizontal lines passing through $y = 7$ and $y = -3$ respectively. The line perpendicular will be vertical. Thus, the distance, is the difference in the y-intercepts of the two lines. Then perpendicular distance between the two horizontal lines is 7 – (–3) = 10 units.

ANSWER:

10 units

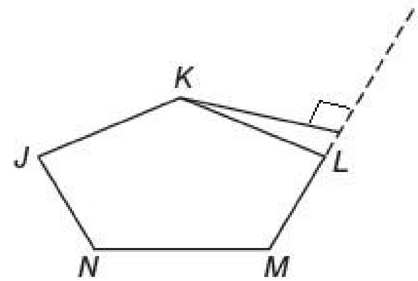
Copy each figure. Construct the segment that represents the distance indicated.

12. K to $\overline{LM}$

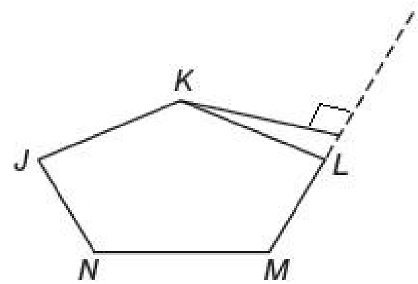


SOLUTION:

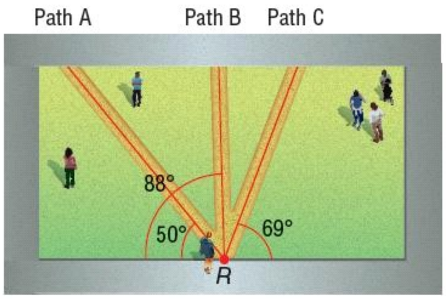
The shortest distance from point K to line $\overline{LM}$ is the length of a segment perpendicular to $\overline{LM}$ from point K. Draw a perpendicular segment from K to $\overline{LM}$.



ANSWER:



14. **APPLY MATH** Rondell is crossing the courtyard in front of his school. Three possible paths are shown in the diagram. Which of the three paths shown is the shortest? Explain your reasoning.



SOLUTION:

The shortest possible distance would be the perpendicular distance from one side of the courtyard to the other. Since Path B is the closest to 90°, it is the shortest of the three paths shown.

ANSWER:

Path B; The shortest possible distance would be the perpendicular distance from one side of the courtyard to the other. Since Path B is the closest to 90°, it is the shortest of the three paths shown.

COORDINATE GEOMETRY Find the distance from *P* to ℓ.

17. Line ℓ contains points (–2, 1) and (4, 1). Point *P* has coordinates (5, 7).

SOLUTION:

Use the slope formula to find the slope of the line ℓ. Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (4, 1)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-1}{4-(-2)} \\ &= \frac{0}{6} \\ &= 0 \end{aligned}$$

Use the slope and any one of the points to write the equation of the line. Let $(x_1, y_1) = (-2, 1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-Slope form} \\ y - 1 &= 0(x-(-2)) && \text{Substitution.} \\ y - 1 &= 0 \\ y - 1 + 1 &= +1 \\ y &= 1 && \text{Equation 1} \end{aligned}$$

The slope of an equation perpendicular to ℓ will be undefined, and hence the line will be a vertical line. The equation of a vertical line through (5, 7) is $x = 5$. The point of intersection of the two lines is (5, 1).

Use the Distance Formula to find the distance between the points (5, 1) and (5, 7). Let $(x_1, y_1) = (5, 1)$ and $(x_2, y_2) = (5, 7)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5-5)^2 + (7-1)^2} \\ &= \sqrt{0 + 36} \\ &= 6 \end{aligned}$$

Therefore, the distance between the line and the point is 6 units.

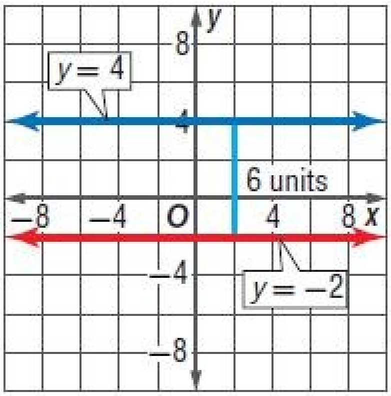
ANSWER:

6 units

Find the distance between each pair of parallel lines with the given equations.

21. $y = -2$
$y = 4$

SOLUTION:



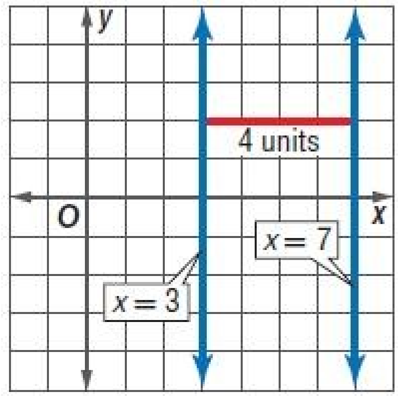
The two lines are horizontal lines and for each equation, the coefficient of x-term is zero. So, the slopes are zero. Therefore, the line perpendicular to the parallel lines will be vertical.

The distance of the vertical line between the parallel lines, will be the difference in the y-intercepts. To find the perpendicular distance between the two horizontal lines subtract –2 from 4 to get 4 – (–2) = 6 units.

ANSWER:
6 units

22. $x = 3$
$x = 7$

SOLUTION:



The two lines are vertical and of the form $x = a$. So, the slopes are undefined. Therefore, the lines are vertical lines passing through $x = 3$ and $x = 7$ respectively. The line perpendicular to each line will be horizontal. The distance will be the difference in the x-intercepts. To find the perpendicular distance between the two horizontal lines subtract 3 from 7 to get 7 – 3 = 4 units

ANSWER:
4 units

23. $y = 5x - 22$
$y = 5x + 4$

SOLUTION:
To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equation, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two point using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.
$y = 5x - 22$ Equation 1
$y = 5x + 4$ Equation 2
The slope of a line perpendicular to both the lines will be $-\frac{1}{5}$. Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line $y = 5x + 4$ is (0,

4). So, the equation of a line with slope $-\frac{1}{5}$ and a y-intercept of 4 is

$$y = -\frac{1}{5}x + 4. \quad \text{Equation 3.}$$

Step 2: Find the intersections of the perpendicular line and each of the other lines.
To find the point of intersection of the perpendicular and the second line, solve the two equations.
The left sides of the equations are the same. So, equate the right sides and solve for x.

$$\begin{aligned}
5x - 22 &= -\frac{1}{5}x + 4 \quad \text{Equation 1} = \text{Equation 3}\\
5x + \frac{1}{5}x - 22 &= -\frac{1}{5}x + \frac{1}{5}x + 4\\
\frac{25}{5}x + \frac{1}{5}x - 22 &= 4\\
\frac{26}{5}x - 22 + 22 &= 4 + 22\\
\frac{26}{5}x &= 26\\
\frac{5}{26}\left(\frac{26}{5}x\right) &= \frac{5}{26}(26)\\
x &= 5 \quad x\text{-coord. of pt. of intersection.}
\end{aligned}$$

Use the value of x to find the value of y.

$$\begin{aligned}
y &= 5x - 22 \quad \text{Equation 1}\\
&= 5(5) - 22\\
&= 25 - 22\\
&= 3 \quad y\text{-coord. of pt. of intersection}
\end{aligned}$$

So, the point of intersection is (5, 3).

Step 3: Find the length of the perpendicular between points
Use the Distance Formula to find the distance between the points (5, 3) and (0, 4). Let (x_1, y_1) = (5, 3) and (x_2, y_2) = (0, 4).

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\\
&= \sqrt{(0-5)^2 + (4-3)^2}\\
&= \sqrt{25 + 1}\\
&= \sqrt{26}
\end{aligned}$$

Therefore, the distance between the two lines is $\sqrt{26}$ units.

ANSWER:

$\sqrt{26}$ units

24. $y = \frac{1}{3}x - 3$

$y = \frac{1}{3}x + 2$

SOLUTION:

To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equation, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two point using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.

$$y = \frac{1}{3}x - 3 \quad \text{Equation 1}$$

$$y = \frac{1}{3}x + 2 \quad \text{Equation 2}$$

The slope of a line perpendicular to both the lines will be –3. Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line $y = \frac{1}{3}x + 2$ is (0, 2). So, the equation of a line with slope –3 and a y-intercept of 2 is

$$y = -3x + 2. \quad \text{Equation 3}$$

Step 2: Find the intersections of the perpendicular line and each of the other lines.
To find the point of intersection of the perpendicular and the second line, solve the two equations.
The left sides of the equations are the same. So, equate the right sides and solve for x.

$$\begin{aligned}
-3x + 2 &= \frac{1}{3}x - 3 \quad \text{Equation 2} = \text{Equation 3}\\
-3x - \frac{1}{3}x + 2 &= \frac{1}{3}x - \frac{1}{3}x - 3\\
-\frac{9}{3}x - \frac{1}{3}x + 2 &= -3\\
-\frac{10}{3}x + 2 - 2 &= -3 - 2\\
\frac{-10}{3}x &= -5\\
-\frac{3}{10}\left(-\frac{10}{3}x\right) &= -\frac{3}{10}(-5)\\
x &= \frac{3}{2}\\
x &= 1\frac{1}{2} \quad x\text{-coord. of pt. of intersection.}
\end{aligned}$$

Use the value of x to find the value of y.

$$
\begin{aligned}
y &= \frac{1}{3}\left(\frac{3}{2}\right) - 3 \quad \text{Equation 1} \\
&= \frac{1}{3}\left(\frac{3}{2}\right) - 3 \\
&= -\frac{5}{2} \\
&= -2\frac{1}{2} \qquad y\text{-coord. of pt. of intersecton}
\end{aligned}
$$

So, the point of intersection is $\left(1\frac{1}{2}, -2\frac{1}{2}\right)$.

Step 3: Find the length of the perpendicular between points.
Use the Distance Formula to find the distance between the points $\left(1\frac{1}{2}, -2\frac{1}{2}\right)$ and (0, 2). Let $(x_1, y_1) = \left(1\frac{1}{2}, -2\frac{1}{2}\right)$ and $(x_2, y_2) = (0, 2)$.

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{\left(0 - \frac{3}{2}\right)^2 + \left(2 - \left(-\frac{5}{2}\right)\right)^2} \\
&= \sqrt{\frac{9}{4} + \frac{81}{4}} \\
&= \sqrt{\frac{90}{4}} \\
&= \frac{3}{2}\sqrt{10}
\end{aligned}
$$

Therefore, the distance between the two lines is $1.5\sqrt{10}$ units.

ANSWER:

$1.5\sqrt{10}$ units