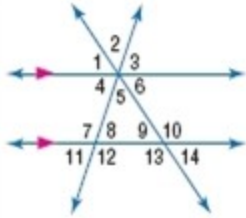


3-2 Angles and Parallel Lines

In the figure, $m\angle 11 = 62$ and $m\angle 14 = 38$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.



12. $\angle 3$

SOLUTION:

In the figure, angles 4 and 11 are corresponding angles and angles 3 and 4 are vertical angles.

$$\angle 4 \cong \angle 11 \quad \text{Corresponding Angles Postulate}$$

$$m\angle 4 = m\angle 11 \quad \text{Definition of congruent angles}$$

$$m\angle 4 = 62 \quad \text{Substitution.}$$

$$\angle 3 \cong \angle 4 \quad \text{Vertical Angles}$$

$$m\angle 3 = m\angle 4 \quad \text{Definition of congruent angles}$$

$$m\angle 3 = 62 \quad \text{Substitution.}$$

ANSWER:

62; Corresponding \angle s Post. and Vertical \angle Thm. or Alt. Ext. Thm.

14. $\angle 8$

SOLUTION:

In the figure, angles 8 and 11 are vertical angles.

$$\angle 8 \cong \angle 11 \quad \text{Vertical Angles}$$

$$m\angle 8 = m\angle 11 \quad \text{Definition of congruent angles}$$

$$m\angle 8 = 62 \quad \text{Substitution.}$$

ANSWER:

62; Vertical Angle Thm.

16. $\angle 2$

SOLUTION:

The angles $\angle 1$ and $\angle 14$ are alternate exterior angles and so are congruent. and angles $\angle 3$ and $\angle 11$ are alternate exterior angles and so are congruent. By Supplementary Theorem, $m\angle 1 + m\angle 2 + m\angle 3 = 180$.

$$\angle 1 \cong \angle 14 \quad \text{Alternate Exterior Angles Theorem}$$

$$m\angle 1 = m\angle 14 \quad \text{Definition of congruent angles}$$

$$m\angle 1 = 38 \quad \text{Substitution.}$$

$$\angle 3 \cong \angle 11 \quad \text{Alternate Exterior Angles Theorem}$$

$$m\angle 3 = m\angle 11 \quad \text{Definition of congruent angles}$$

$$m\angle 3 = 62 \quad \text{Substitution.}$$

$$\angle 1 + \angle 2 + \angle 3 \cong 180^\circ \quad \text{Def. of supplementary angles}$$

$$m\angle 1 + m\angle 2 + m\angle 3 = 180 \quad \text{Def. of congruent angles}$$

$$38 + m\angle 2 + 62 = 180 \quad \text{Substitution.}$$

$$100 + m\angle 2 = 180 \quad \text{Simplify.}$$

$$100 - 100 + m\angle 2 = 180 - 100 \quad \text{Subtract 100 from each side.}$$

$$m\angle 2 = 80 \quad \text{Simplify.}$$

ANSWER:

80; Alt. Ext. \angle s Post. and Supp. \angle Thm.

18. $\angle 5$

SOLUTION:

Use definition of supplementary angles, Corresponding Angles Postulate and the Alternate Interior Angles Theorem .

$$\angle 11 + \angle 7 \cong 180^\circ \quad \text{Definition of supplementary angles}$$

$$m\angle 11 + m\angle 7 = 180 \quad \text{Def. of congruent angles}$$

$$62 + m\angle 7 = 180 \quad \text{Substitution.}$$

$$62 - 62 + m\angle 7 = 180 - 62 \quad \text{Subtract 62 from each side.}$$

$$m\angle 7 = 118 \quad \text{Simplify.}$$

$$\angle 6 \cong \angle 14 \quad \text{Corresponding Angles Postulate}$$

$$m\angle 6 = m\angle 14 \quad \text{Definition of congruent angles}$$

$$m\angle 6 = 38 \quad \text{Substitution.}$$

$$\angle 7 \cong \angle 5 + \angle 6 \quad \text{Alternate Interior Angles Theorem}$$

$$m\angle 7 = m\angle 5 + m\angle 6 \quad \text{Definition of congruent angles}$$

$$118 = m\angle 5 + 38 \quad \text{Substitution.}$$

$$118 - 38 = m\angle 5 + 38 - 38 \quad \text{Subtract 38 from each side.}$$

$$80 = m\angle 5 \quad \text{Substitution.}$$

ANSWER:

80; Vertical Angles Thm.

3-2 Angles and Parallel Lines

APPLY MATH A solar dish collects energy by directing radiation from the Sun to a receiver located at the focal point of the dish. Assume that the radiation rays are parallel. Determine the relationship between each pair of angles, and explain your reasoning.

Refer to Page 183.

20. $\angle 1$ and $\angle 2$

SOLUTION:

If the radiation rays form parallel lines, then $\angle 1$ and $\angle 2$ are consecutive interior angles. So, according to the Consecutive Interior Angles Theorem, $\angle 1$ and $\angle 2$ are supplementary.

ANSWER:

supplementary; Consecutive Interior Angles

21. $\angle 1$ and $\angle 3$

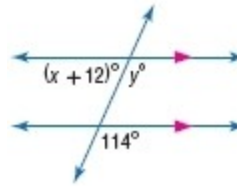
SOLUTION:

If the radiation rays form parallel lines, then $\angle 1$ and $\angle 3$ are corresponding angles. So, according to the Corresponding Angles Postulate, $\angle 1$ and $\angle 3$ are congruent.

ANSWER:

congruent; Corresponding Angles

Find the value of the variable(s) in each figure. Explain your reasoning.



24.

SOLUTION:

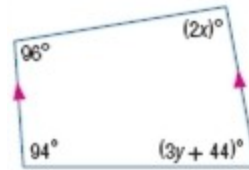
Use Corresponding Angles Postulate and definition of supplementary angles to find x .

$$m\angle y = 114 \quad \text{Corresponding Angles Postulate}$$

$$\begin{aligned} m\angle y + (x + 12) &= 180 && \text{Definition of supplementary angles} \\ 114 + x + 12 &= 180 && \text{Substitution.} \\ 126 + x &= 180 && \text{Simplify.} \\ 126 - 126 + x &= 180 - 126 && \text{Subtract 126 from each side.} \\ x &= 54 && \text{Simplify.} \end{aligned}$$

ANSWER:

$y = 114$ by the Corresponding Angles Postulate; $x = 54$ by the Supplement Theorem



27.

SOLUTION:

Use the Consecutive Interior Angles Theorem to find x and y .

$$\begin{aligned} (2x)^\circ + 96^\circ &\cong 180^\circ && \text{Consecutive Interior Angles Theorem} \\ 2x + 96 &= 180 && \text{Definition of congruent angles} \\ 2x + 96 - 96 &= 180 - 96 && \text{Subtract 96 from each side.} \\ 2x &= 84 && \text{Simplify.} \\ \frac{2x}{2} &= \frac{84}{2} && \text{Divide each side by 2.} \\ x &= 42 && \text{Simplify.} \end{aligned}$$

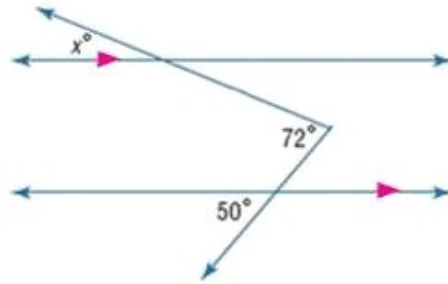
$$\begin{aligned} 94^\circ + (3y + 44)^\circ &\cong 180^\circ && \text{Consecutive Interior Angles Theorem} \\ 94 + 3y + 44 &= 180 && \text{Definition of congruent angles} \\ 3y + 138 &= 180 && \text{Simplify.} \\ 3y + 138 - 138 &= 180 - 138 && \text{Subtract 138 from each side.} \\ 3y &= 42 && \text{Simplify.} \\ \frac{3y}{3} &= \frac{42}{3} && \text{Divide each side by 3.} \\ y &= 14 && \text{Simplify.} \end{aligned}$$

ANSWER:

$x = 42$ by the Consecutive Interior Angles Theorem; $y = 14$ by the Consecutive Interior Angles Theorem

3-2 Angles and Parallel Lines

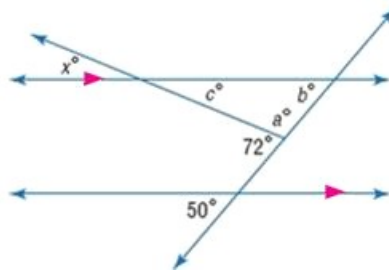
TOOLS AND TECHNIQUES Find x . (*Hint: Draw an auxiliary line.*)



38.

SOLUTION:

Draw an auxiliary line to construct a triangle. Then label the angles a° , b° , and c° . By finding the measures for angles a and b , we can use the Triangle Angle Sum theorem to find angle c . Angles c and x are vertical angles.



Use the definition of supplementary angles to find a .

$$\begin{aligned} 72^\circ + a^\circ &\cong 180^\circ && \text{Def. of supplementary angles} \\ 72 + a &= 180 && \text{Def. of congruent angles} \\ 72 - 72 + a &= 180 - 72 && \text{Subtract 72 from each side.} \\ a &= 108 && \text{Simplify.} \end{aligned}$$

Find angle b .

$$\begin{aligned} b^\circ &\cong 50^\circ && \text{Corresponding Angles Theorem} \\ b &= 50 && \text{Definition of congruent angles} \end{aligned}$$

Find angle c .

$$\begin{aligned} a^\circ + b^\circ + c^\circ &\cong 180^\circ && \text{Triangle Angle Sum Theorem} \\ a + b + c &= 180 && \text{Definition of congruent angles} \\ 108 + 50 + c &= 180 && \text{Substitution.} \\ 158 + c &= 180 && \text{Simplify.} \\ 158 - 158 + c &= 180 - 158 && \text{Subtract 158 from each side.} \\ c &= 22 && \text{Simplify.} \end{aligned}$$

Find angle x .

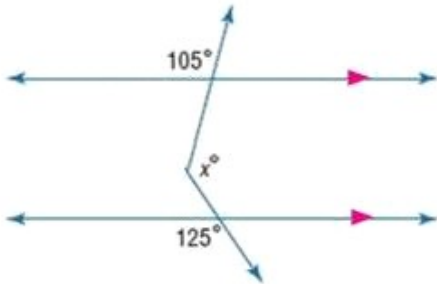
$$\begin{aligned} c^\circ &\cong x^\circ && \text{Vertical Angles} \\ c &= x && \text{Def. of congruent angles} \\ 22 &= x && \text{Substitution.} \end{aligned}$$

So, $x = 22$.

ANSWER:

22

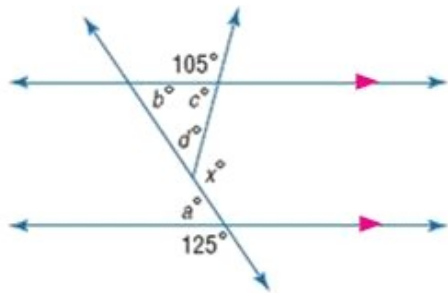
3-2 Angles and Parallel Lines



39.

SOLUTION:

Draw an auxiliary line to construct a triangle. By creating a triangle, we can use the Triangle Angle Sum Theorem and definition of supplementary angles to find x . Label the angles.



First find angle a .

$$\begin{aligned} a + 125^\circ &\cong 180^\circ && \text{Def. of supplementary angles} \\ a + 125 &= 180 && \text{Def. of congruent angles} \\ a + 125 - 125 &= 180 - 125 && \text{Subtract 125 from each side.} \\ a &= 55 && \text{Simplify.} \end{aligned}$$

Find angle b .

$$\begin{aligned} a^\circ &\cong b^\circ && \text{Alternate Interior Angles Theorem} \\ a &= b && \text{Definition of congruent angles} \\ 55 &= b && \text{Substitution.} \end{aligned}$$

Find angle c .

$$\begin{aligned} c + 105^\circ &\cong 180^\circ && \text{Def. of supplementary angles} \\ c + 105 &= 180 && \text{Def. of congruent angles} \\ c + 105 - 105 &= 180 - 105 && \text{Subtract 105 from each side.} \\ c &= 75 && \text{Simplify.} \end{aligned}$$

Find angle d .

$$\begin{aligned} b^\circ + c^\circ + d^\circ &\cong 180^\circ && \text{Triangle Angle Sum Theorem} \\ b + c + d &= 180 && \text{Def. of congruent angles} \\ 55 + 75 + d &= 180 && \text{Substitution.} \\ 130 + d &= 180 && \text{Simplify.} \\ 130 - 130 + d &= 180 - 130 && \text{Subtract 130 from each side.} \\ d &= 50 && \text{Simplify.} \end{aligned}$$

Find angle x

$$\begin{aligned} x^\circ + d^\circ &\cong 180^\circ && \text{Def. of supplementary angles} \\ x + d &= 180 && \text{Def. of congruent angles} \\ x + 50 &= 180 && \text{Substitution.} \\ x + 50 - 50 &= 180 - 50 && \text{Subtract 50 from each side.} \\ x &= 130 && \text{Simplify.} \end{aligned}$$

So $x = 130^\circ$.

ANSWER:

130