Algebraic Proofs

Proving Segment and Angle Relationships

Algebraic Proof

- An algebraic proof is a proof that is made up of a series of algebraic statements.
- The properties of equality provide justification for many statements in algebraic proofs.

Properties of Real Numbers

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then, $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

Examples

- Prove that if $-5(x+4)=70$, then $x=-18$. Write a justification for each step.

Other Properties

- Commutative Property of Addition

  $a + b = b + a$
- Commutative Property of Multiplication

  $a * b = b * a$
- Associative Property of Addition

  $(a + b) + c = a + (b + c)$
- Associative Property of Multiplication

  $(a * b) * c = a * (b * c)$

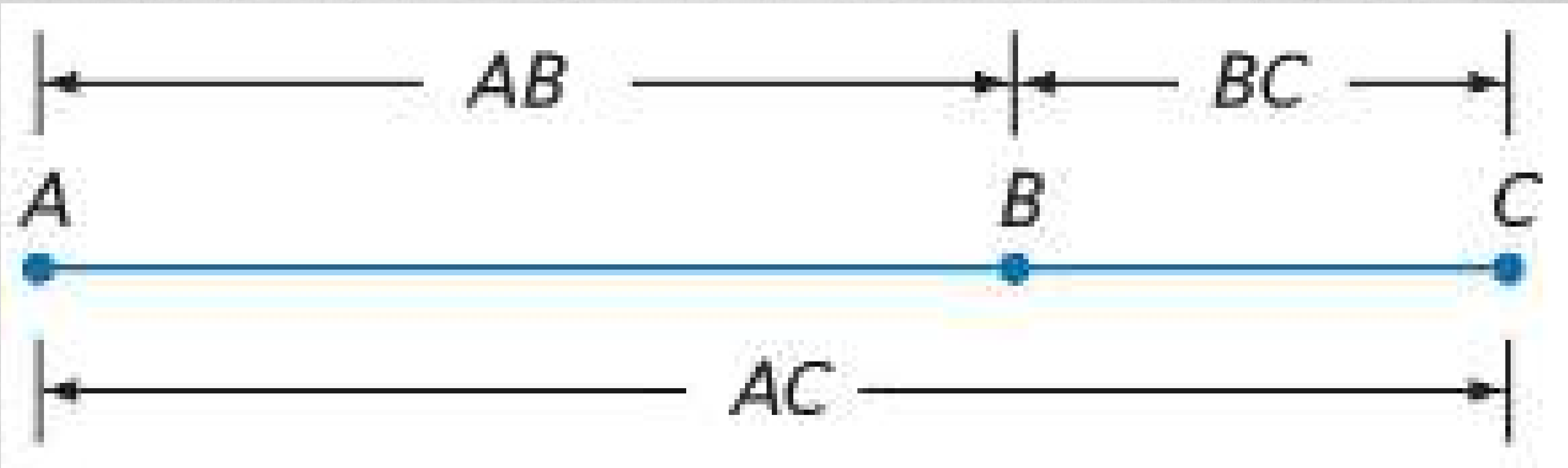
Geometric Proof

- A geometric proof is the same as an algebraic proof, with the exception of using geometric concepts versus algebraic concepts.
- Since geometry also uses variables, numbers, and operations, many of the properties of equality used in algebra are also true in geometry.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

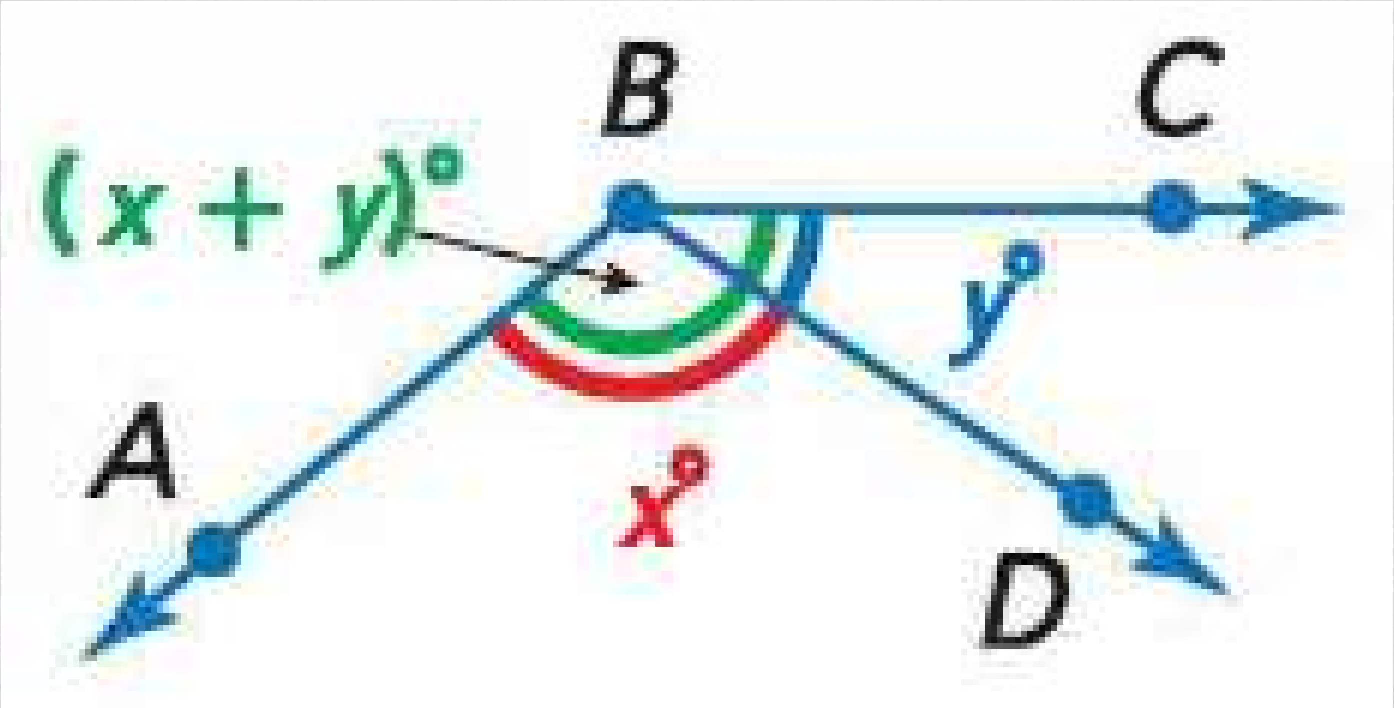
Segment Addition Postulate

- If A, B, and C are collinear, then point B is between A and C if and only if AB+BC=AC



Angle Addition Postulate

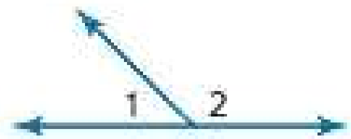
- D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$



Relevant Theorems

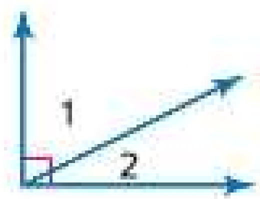
Theorems

2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.



Example $m\angle 1 + m\angle 2 = 180$

2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

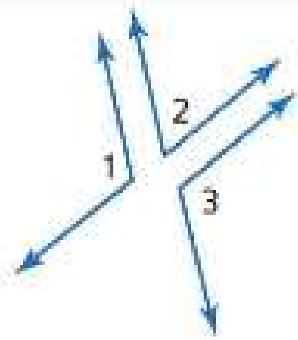


Example $m\angle 1 + m\angle 2 = 90$

Relevant Theorems

Theorems

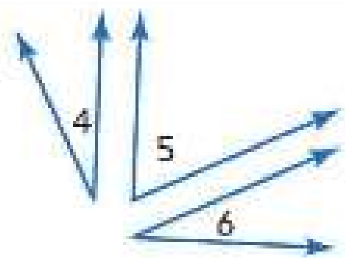
2.6 Congruent Supplements Theorem
Angles supplementary to the same angle or to congruent angles are congruent.



Abbreviation *∡ suppl. to same ∠ or ≅ ∡ are ≅.*

Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.

2.7 Congruent Complements Theorem
Angles complementary to the same angle or to congruent angles are congruent.



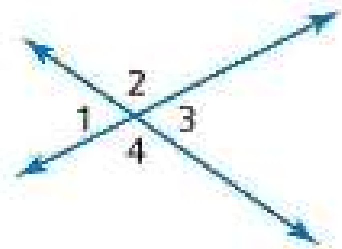
Abbreviation *∡ compl. to same ∠ or ≅ ∡ are ≅.*

Example If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.

Relevant Theorems

Theorem 2.8 Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.

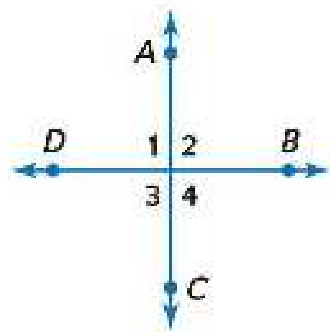
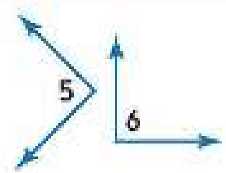
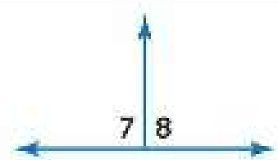


Abbreviation *Vert.* $\angle$s *are* $\cong$.

Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

Relevant Theorems

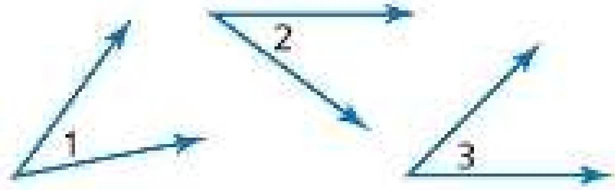
Theorems Right Angle Theorems

Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s.	
2.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. $\angle$s.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. $\angle$s.	

Examples

Given: ∠1 and ∠3 are complementary.
∠2 and ∠3 are complementary.

Prove: ∠1 ≅ ∠2



Proof:

Statements	Reasons
a. ∠1 and ∠3 are complementary. ∠2 and ∠3 are complementary.	**a.** __?__
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** __?__
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** __?__
d. __?__	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	e. __?__
f. ∠1 ≅ ∠2	**f.** __?__

Examples

Given: $4x + 8 = x + 2$

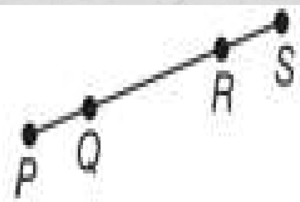
Prove: $x = -2$

Proof:

Statements	Reasons
a. $4x + 8 = x + 2$	**a.**______________________
b. $4x + 8 - x = x + 2 - x$	**b.**______________________
c. $3x + 8 = 2$	**c.** Substitution
d.______________________	**d.** Subtr. Prop.
e.______________________	**e.** Substitution
f. $\frac{3x}{3} = \frac{-6}{3}$	**f.**______________________
g.______________________	**g.** Substitution

Examples

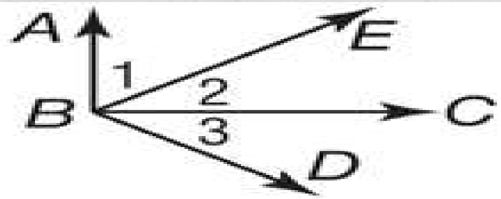
Given: $\overline{PR} \cong \overline{QS}$
Prove: $\overline{PQ} \cong \overline{RS}$



Proof:

Statements	Reasons
a. $\overline{PR} \cong \overline{QS}$	a. ______________________
b. $PR = QS$	b. ______________________
c. $PQ + QR = PR$	c. ______________________
d. ______________________	d. Segment Addition Postulate
e. $PQ + QR = QR + RS$	e. ______________________
f. ______________________	f. Subtraction Property
g. ______________________	g. Definition of congruence of segments

Examples



Given: $\overline{AB} \perp \overline{BC}$;
$\angle 1$ and $\angle 3$ are complementary.
Prove: $\angle 2 \cong \angle 3$
Proof:

Statements	Reasons
a. $\overline{AB} \perp \overline{BC}$	a. ________
b. ________	b. Definition of $\perp$
c. $m\angle ABC = 90$	c. Def. of right angle
d. $m\angle ABC = m\angle 1 + m\angle 2$	d. ________
e. $90 = m\angle 1 + m\angle 2$	e. Substitution
f. $\angle 1$ and $\angle 2$ are compl.	f. ________
g. ________	g. Given
h. $\angle 2 \cong \angle 3$	h. ________